Outline

• Introduction
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• Acceleration Field
• Control Volume and System Representation
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Fluid Kinematics: Introduction

- Fluids subject to shear, flow
- Fluids subject to pressure imbalance, flow
- In kinematics we are not concerned with the force, but the motion.
- Thus, we are interested in visualization.
- We can learn a lot about flows from watching.
Continuum Hypothesis: the flow is made of tightly packed fluid particles that interact with each other. Each particle consists of numerous molecules, and we can describe velocity, acceleration, pressure, and density of these particles at a given time.

\[ \mathbf{V} = u(x, y, z, t) \mathbf{i} + v(x, y, z, t) \mathbf{j} + w(x, y, z, t) \mathbf{k} \]

\[ \mathbf{V} = \mathbf{V}(x, y, z, t) \]

\[ V = |\mathbf{V}| = (u^2 + v^2 + w^2)^{1/2} \]

\[ \frac{d\mathbf{r}_A}{dt} = \mathbf{V}_A \]
**Velocity Field: Eulerian and Lagrangian**

**Eulerian:** the fluid motion is given by completely describing the necessary properties as a function of space and time. We obtain information about the flow by noting what happens at fixed points.

**Lagrangian:** following individual fluid particles as they move about and determining how the fluid properties of these particles change as a function of time.

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**Measurement of Temperature**

- **Eulerian:**
  - Location 0: \( T = T(x_0, y_0, t) \)
  - Particle A: \( T_A = T_A(t) \)

- **Lagrangian:**

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If we have enough information, we can obtain Eulerian from Lagrangian or vice versa.

Eulerian methods are commonly used in fluid experiments or analysis—a probe placed in a flow.

Lagrangian methods can also be used if we “tag” fluid particles in a flow.
Velocity Field: 1D, 2D, and 3D Flows

Most fluid flows are complex three dimensional, time-dependent phenomenon, however we can make simplifying assumptions allowing an easier analysis or understanding without sacrificing accuracy. In many cases we can treat the flow as 1D or 2D flow.

Three-Dimensional Flow: All three velocity components are important and of equal magnitude. Flow past a wing is complex 3D flow, and simplifying by eliminating any of the three velocities would lead to severe errors.

Two-Dimensional Flow: In many situations one of the velocity components may be small relative to the other two, thus it is reasonable in this case to assume 2D flow.

One-Dimensional Flow: In some situations two of the velocity components may be small relative to the other one, thus it is reasonable in this case to assume 1D flow. There are very few flows that are truly 1D, but there are a number where it is a reasonable approximation.
Velocity Field: **Steady and Unsteady Flows**

**Steady Flow:** The velocity at a given point in space does not vary with time.

\[ \frac{\partial V}{\partial t} = 0 \]

Very often, we assume steady flow conditions for cases where there is only a slight time dependence, since the analysis is “easier”

**Unsteady Flow:** The velocity at a given point in space does vary with time.

Almost all flows have some unsteadiness. In addition, there are periodic flows, non-periodic flows, and completely random flows.

**Examples:**

- **Nonperiodic flow:** “water hammer” in water pipes.
- **Periodic flow:** “fuel injectors” creating a periodic swirling in the combustion chamber. Effect occurs time after time.
- **Random flow:** “Turbulent”, gusts of wind, splashing of water in the sink

Steady or Unsteady only pertains to fixed measurements, i.e. exhaust temperature from a tail pipe is relatively constant “steady”; however, if we followed individual particles of exhaust they cool!
Streamline: the line that is everywhere tangent to the velocity field. If the flow is steady, nothing at a fixed point changes in time. In an unsteady flow the streamlines due change in time.

Analytically, for 2D flows, integrate the equations defining lines tangent to the velocity field:

\[
\frac{dy}{dx} = \frac{v}{u}
\]

Experimentally, flow visualization with dyes can easily produce the streamlines for a steady flow, but for unsteady flows these types of experiments don’t necessarily provide information about the streamlines.
**Velocity Field: Streaklines**

**Streaklines:** a laboratory tool used to obtain instantaneous photographs of marked particles that all passed through a given flow field at some earlier time. Neutrally buoyant smoke, or dye is continuously injected into the flow at a given location to create the picture.

If the flow is steady, the picture will look like streamlines (previous video).

If the flow is unsteady, the picture will be of the instantaneous flow field, but will change from frame to frame, “streaklines”.

![Image of streaklines experiment](Image)
**Velocity Field: Pathlines**

**Pathlines:** line traced by a given particle as it flows from one point to another. This method is a Lagrangian technique in which a fluid particle is marked and then the flow field is produced by taking a time exposure photograph of its movement.

If the flow is steady, the picture will look like streamlines (previous video).

If the flow is unsteady, the picture will be of the instantaneous flow field, but will change from frame to frame, “pathlines”.
Acceleration Field

Lagrangian Frame: \( \mathbf{a} = \mathbf{a}(t) \)

Eulerian Frame: we describe the acceleration in terms of position and time without following an individual particle. This is analogous to describing the velocity field in terms of space and time.

\[
\mathbf{V}_A = \mathbf{V}_A(\mathbf{r}_A, t) = \mathbf{V}_A[x_A(t), y_A(t), z_A(t), t]
\]

A fluid particle can accelerate due to a change in velocity in time ("unsteady") or in space (moving to a place with a greater velocity).
Acceleration Field: Material (Substantial) Derivative

\[ \mathbf{a}_A(t) = \frac{d\mathbf{V}_A}{dt} = \frac{\partial \mathbf{V}_A}{\partial t} + \frac{\partial \mathbf{V}_A}{\partial x} \frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y} \frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z} \frac{dz_A}{dt} \]

We note:

\[ u_A = \frac{dx_A}{dt}, \quad v_A = \frac{dy_A}{dt}, \quad w_A = \frac{dz_A}{dt} \]

Then, substituting:

\[ \mathbf{a}_A = \frac{\partial \mathbf{V}_A}{\partial t} + u_A \frac{\partial \mathbf{V}_A}{\partial x} + v_A \frac{\partial \mathbf{V}_A}{\partial y} + w_A \frac{\partial \mathbf{V}_A}{\partial z} \]

The above is good for any fluid particle, so we drop “A”:

\[ \mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y} + w \frac{\partial \mathbf{V}}{\partial z} \]
Acceleration Field: Material (Substantial) Derivative

Writing out these terms in vector components:

x-direction: \( a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \)

y-direction: \( a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \)

z-direction: \( a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \)

Writing these results in “short-hand”: \( \mathbf{a} = \frac{D\mathbf{V}}{Dt} \)

Fluid flows experience fairly large accelerations or decelerations, especially approaching stagnation points.

where,

\[ \frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u \frac{\partial(\ )}{\partial x} + v \frac{\partial(\ )}{\partial y} + w \frac{\partial(\ )}{\partial z} \]

\[ \frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V} \cdot \nabla)(\ ) \]

\( \nabla(\ ) = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \), \( \mathbf{V} \cdot \nabla(\ ) = u \frac{\partial(\ )}{\partial x} + v \frac{\partial(\ )}{\partial y} + w \frac{\partial(\ )}{\partial z} \)
Acceleration Field: Material (Substantial) Derivative

Applied to the Temperature Field in a Flow:

\[ T = T(x, y, z, t) \]
\[ \mathbf{V} = \mathbf{V}(x, y, z, t) \]

The material derivative of any variable is the rate at which that variable changes with time for a given particle (as seen by one moving along with the fluid—Lagrangian description).

\[
\frac{dT_A}{dt} = \frac{\partial T_A}{\partial t} + \frac{\partial T_A}{\partial x} \frac{dx_A}{dt} + \frac{\partial T_A}{\partial y} \frac{dy_A}{dt} + \frac{\partial T_A}{\partial z} \frac{dz_A}{dt}
\]

\[
\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T
\]
Acceleration Field: Unsteady Effects

If the flow is unsteady, its parameter values at any location may change with time (velocity, temperature, density, etc.)

The local derivative represents the unsteady portion of the flow: \[ \frac{\partial}{\partial t} \]

If we are talking about velocity, then the above term is local acceleration.

In steady flow, the above term goes to zero.

If we are talking about temperature, and \( \mathbf{V} = 0 \), we still have heat transfer because of the following term:

\[
\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u \left( \frac{\partial T}{\partial x} \right)_0 + v \left( \frac{\partial T}{\partial y} \right)_0 + w \left( \frac{\partial T}{\partial z} \right)_0
\]

\[ = \frac{\partial T}{\partial t} \]
Consider flow in a constant diameter pipe, where the flow is assumed to be spatially uniform: \( \mathbf{V} = V_0(t) \mathbf{\hat{i}} \)

\[
\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + \phi \frac{\partial \mathbf{V}}{\partial y} + w \frac{\partial \mathbf{V}}{\partial z}
\]

\[
\frac{\partial V_0}{\partial t} \mathbf{\hat{i}}
\]
Acceleration Field: Convective Effects

The portion of the material derivative represented by the spatial derivatives is termed the convective term or convective acceleration: \( (\nabla \cdot \mathbf{V}) \mathbf{V} \).

It represents the fact the flow property associated with a fluid particle may vary due to the motion of the particle from one point in space to another.

Convective effects may exist whether the flow is steady or unsteady.

Example 1:

Example 2:

\[
\frac{a_x}{u_1} = \frac{\partial u}{\partial x}
\]

\( \frac{\partial u}{\partial x} > 0 \quad a_x > 0 \)

\( \frac{\partial u}{\partial x} < 0 \quad a_x < 0 \)

Acceleration = Deceleration
**Control Volume and System Representations**

**Systems of Fluid:** a specific identifiable quantity of matter that may consist of a relatively large amount of mass (the earth’s atmosphere) or a single fluid particle. They are always the same fluid particles which may interact with their surroundings.

Example: following a system the fluid passing through a compressor

We can apply the equations of motion to the fluid mass to describe their behavior, but in practice it is very difficult to follow a specific quantity of matter.

**Control Volume:** is a volume or space through which the fluid may flow, usually associated with the geometry.

When we are most interested in determining the forces put on a fan, airplane, or automobile by the air flow past the object rather than following the fluid as it flows along past the object.

Identify the specific volume in space and analyze the fluid flow within, through, or around that volume.
Control Volume and System Representations

Fixed Control Volume:

Fixed or Moving Control Volume:

Deforming Control Volume:
Reynolds Transport Theorem: Preliminary Concepts

All the laws of governing the motion of a fluid are stated in their basic form in terms of a system approach, and not in terms of a control volume.

The Reynolds Transport Theorem allows us to shift from the system approach to the control volume approach, and back.

General Concepts: \( B = mb \)

\( B \) represents any of the fluid properties, \( m \) represent the mass, and \( b \) represents the amount of the parameter per unit volume.

Examples:

- **Mass** \( b = 1 \)
- **Kinetic Energy** \( b = \frac{V^2}{2} \)
- **Momentum** \( b = V \) (vector)

\( B \) is termed an extensive property, and \( b \) is an intensive property. \( B \) is directly proportional to mass, and \( b \) is independent of mass.
Reynolds Transport Theorem: Preliminary Concepts

For a System: The amount of an extensive property can be calculated by adding up the amount associated with each fluid particle.

\[ B_{sys} = \lim_{\delta V \to 0} \sum_{i} b_{i}(\rho_{i} \delta V_{i}) = \int_{sys} \rho b \, dV \]

Now, the time rate of change of that system:

\[ \frac{dB_{sys}}{dt} = \frac{d}{dt} \left( \int_{sys} \rho b \, dV \right) \]

Now, for control volume:

\[ \frac{dB_{cv}}{dt} = \frac{d}{dt} \left( \int_{cv} \rho b \, dV \right) \]

For the control volume, we only integrate over the control volume, this is different integrating over the system, though there are instance when they could be the same.
Consider a 1D flow through a fixed control volume between (1) and (2):

\[ \delta \ell_1 = V_1 \delta t \]
\[ \delta \ell_2 = V_2 \delta t \]

Writing equation in terms of the extensive parameter:

Originally, \( B_{sys}(t) = B_{cv}(t) \)

At time 2:
\[ B_{sys}(t + \delta t) = B_{cv}(t + \delta t) - B_I(t + \delta t) + B_{II}(t + \delta t) \]

Divide by \( \delta t \):
\[ \frac{\delta B_{sys}}{\delta t} = \frac{B_{sys}(t + \delta t) - B_{sys}(t)}{\delta t} = \frac{B_{cv}(t + \delta t) - B_I(t + \delta t) + B_{II}(t + \delta t) - B_{sys}(t)}{\delta t} \]
Reynolds Transport Theorem: Derivation

Noting, $B_{sys}(t) = B_{cv}(t)$

$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{cv}(t + \delta t) - B_{cv}(t)}{\delta t} - \frac{B_1(t + \delta t)}{\delta t} + \frac{B_{II}(t + \delta t)}{\delta t}$$

Let, $\delta t \to 0$

(1) $\frac{\delta B_{sys}}{\delta t} \to DB_{sys}/Dt$

Time rate of change of mass within the control volume:

(2) $\frac{B_{cv}(t + \delta t) - B_{cv}(t)}{\delta t} \to \lim_{\delta t \to 0} \frac{B_{cv}(t + \delta t) - B_{cv}(t)}{\delta t} = \frac{\partial B_{cv}}{\partial t} = \frac{\partial}{\partial t} \left( \int_{cv} \rho b \, dV \right)$

The rate at which the extensive property flows out of the control surface:

(4) $B_{II}(t + \delta t) = (\rho_2 b_2) (\delta V_{II}) = \rho_2 b_2 A_2 V_2 \delta t$

$\delta V_{II} = A_2 \delta \ell_2 = A_2 (V_2 \delta t)$

$\hat{B}_{out} = \lim_{\delta t \to 0} \frac{B_{II}(t + \delta t)}{\delta t} = \rho_2 A_2 V_2 b_2$
Reynolds Transport Theorem: Derivation

The rate at which the extensive property flows into the control surface:

\[ B_1(t + \delta t) = (\rho_1 b_1)(\delta V_1) = \rho_1 b_1 A_1 V_1 \delta t \]

\[ \delta V_1 = A_1 \delta \ell_1 = A_1(V_1 \delta t) \]

\[ \dot{B}_{in} = \lim_{\delta t \to 0} \frac{B_1(t + \delta t)}{\delta t} = \rho_1 A_1 V_1 b_1 \]

Now, collecting the terms:

\[ \frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \dot{B}_{out} - \dot{B}_{in} \]

or

\[ \frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1 \]

Restrictions for the above Equation:

1) Fixed control volume
2) One inlet and one outlet
3) Uniform properties
4) Normal velocity to section (1) and (2)
Reynolds Transport Theorem: Derivation

The Reynolds Transport Theorem can be derived for more general conditions.

**Result:**

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{CV} \rho b \, dV + \int_{CS} \rho b \, \mathbf{V} \cdot \mathbf{\hat{n}} \, dA$$

This form is for a fixed non-deforming control volume.
Reynolds Transport Theorem: Physical Interpretation

\[
\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b \, dV + \int_{cs} \rho b \, \mathbf{V} \cdot \mathbf{n} \, dA
\]

(1) The time rate of change of the extensive parameter of a system, mass, momentum, energy.

(2) The time rate of change of the extensive parameter within the control volume.

(3) The net flow rate of the extensive parameter across the entire control surface.

\[
\begin{align*}
\mathbf{V} \cdot \mathbf{n} > 0 & \quad \text{“outflow across the surface”} \\
\mathbf{V} \cdot \mathbf{n} < 0 & \quad \text{“inflow across the surface”} \\
b\mathbf{V} \cdot \mathbf{n} = 0 & \quad \text{“no flow across the surface”}
\end{align*}
\]

\[
b = 0 \quad \mathbf{V} = 0 \\
\mathbf{V} \text{ is parallel}
\]

Mass flow rate: \( \rho \mathbf{V} \cdot \mathbf{n} \, \delta A \)
Reynolds Transport Theorem: Analogous to Material Derivative

\[
\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b \, dV + \int_{cs} \rho b \, \mathbf{V} \cdot \mathbf{n} \, dA
\]

Unsteady Portion

Convective Portion

Steady Effects:

\[
\frac{DB_{sys}}{Dt} = \int_{cs} \rho b \mathbf{V} \cdot \mathbf{n} \, dA
\]

Unsteady Effects (inflow = outflow):

\[
\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b \, dV
\]
Reynolds Transport Theorem: Moving Control Volume

There are cases where it is convenient to have the control volume move. The most convenient is when the control volume moves with a constant velocity.

\[ V_{CV} = V - W \quad \text{relative velocity, } W \]
\[ V = W + V_{CV} \]

\[ V_0 = 20i \text{ ft/s}, \quad V_1 = 100i \text{ ft/s} \], Then \( W = 80i \text{ ft/s} \)

Now, in general for a constant velocity control volume:

\[
\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho b \, dV + \int_{cs} \rho b \ W \cdot \hat{n} \ dA
\]
Reynolds Transport Theorem: Choosing a Control Volume

If we want to know a property at point 1, pressure or velocity for instance:

- Good choice, since the point we want to know is on control surface. Likewise, the values at the inlet and exit are normal to the surface.

- Valid control volume, but the point we want to know is interior. So, it unlikely we will have enough information to obtain its value.

- Valid control volume, but the surfaces are not normal to the inlet and outlet.
Some Example Problems