Experimental study of characteristic curves of centrifugal pumps working as turbines in different specific speeds

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Received 11 February 2007; received in revised form 30 September 2007; accepted 2 October 2007

Abstract

Pump manufacturers do not normally provide the characteristic curves of their pumps working as turbines. Therefore, establishing a correlation between the performances of direct (pump) and reverse (turbine) modes is essential in selecting the proper machine.

In this paper, several centrifugal pumps ($N_s < 60$ (m, m$^3$/s)) were tested as turbines. Using experimental data, some relations were derived to predict the best efficiency point of a pump working as a turbine, based on pump hydraulic characteristics. Validity of the presented method was shown using some referenced experimental data.

Two equations were presented to estimate the complete characteristic curves of centrifugal pumps as turbines based on their best efficiency point. Deviation of suggested method from experimental data were considered and discussed. Finally, a procedure was presented for selecting a suitable pump to work as a turbine in a small hydro-site.

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Keywords: Pump as turbine; Best efficiency point; Characteristic curves; Specific speed; Small hydro-site

1. Introduction

Small hydropower stations became attractive after the oil price crisis of the 70s and again in recent years. However, the cost-per-kW of the energy produced by these stations is higher than the large hydroelectric power plants. In recent years, numerous publications emphasized the importance of using simple turbines to reduce the cost of the generated energy.

In many developing countries, the small hydropower stations are in demand. For example, a feasibility study showed that more than 200 small hydropower stations can be installed in Iran [1]. Using pump as turbine (PAT) is an attractive and significant alternative. Pumps are relatively simple machines with no special designing and are readily available in most developing countries. Besides, their installation, commissioning and maintenance are easy and cheap. From the economical point of view, it is often stated that capital payback period of PATs in the range of 5–500 kW is two years or less [1].

Nowadays, applications of PAT have been developed in villages, farms, irrigation systems, as pressure dropping valves and as small pump storage power stations. Since 1930, the main challenge in PAT usage was the selection of a proper PAT for a small hydro-site.

Some researchers presented relations between pump’s and turbine’s best efficiency points (BEP) based on experimental data and theoretical analyses [2–12]. The results obtained by these relations had almost ±20% deviation from experimental data [9]. Most recent attempts to predict performance of PAT were made using computational fluid dynamics (CFD) [14]. However, without verifying the CFD results by experimental data, they are not reliable.

In this paper, some relations were derived to predict BEP of PATs based on experiments done on several low-specific-speed centrifugal PATs. Besides, a method was...
suggested to derive complete characteristic curves in part-
load and over-load zones. Deviations from experimental
data and comparison with other relations were also pre-
sented. Finally, a procedure was presented to select a
proper PAT for a small hydro-site.

2. PAT field applications

The field applications of conventional turbines such as
Pelton, Francis and Kaplan are well known according to
their heads and specific speeds. Due to inadequate experi-
mental data for pumps working as turbines, the field appli-
cations of these machines are not yet well defined.
However, the field applications of multistage, single impel-
lcr centrifugal and axial pumps can be compared with Pel-
ton, Francis and Kaplan turbines, respectively.

3. Experimental setup

A complete model of a mini hydropower plant was
installed in University of Tehran as shown in Fig. 1. The
flow rate and head for each PAT were generated by a
proper pump in the experimental setup.

Every PAT needs an automatic frequency regulator. The
typical governors used for standard turbines are expensive
and not always recommended for small hydropower plants.
Since these types of plants are usually being installed in
remote areas, using an electronic load controller with bal-
last loads is common. In the test rig, an electronic load con-
troller with ballast loads was built and used for keeping the
frequency of a conventional synchronous generator.

To measure the turbine’s shaft torque, the generator was
modified to suspense state mode and a scaled arm and sev-
eral weights were used (Fig. 2). The flow rate was measured
at discharge pipe using a specific orifice plate for each test.
Pressure was measured by barometers ranging between 0
and 5 bar.

Four industrial centrifugal pumps with specific speeds
from 14 to 56 (m,m$^3$/s) were selected for experiments.
These pumps had input power, head and flow rate up to
30 kW, 25 m and 0.15 m$^3$/s, respectively. To test each
PAT, a feed pump, several pipes, an orifice, a generator

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**Nomenclature**

- $D$: impeller’s nominal diameter, m
- $g$: gravitational acceleration, m/s$^2$
- $H$: head, m
- $h$: head ratio
- $N$: rotational speed, rpm
- $n$: rotational speed, rps
- $N_s$: specific speed
- $P$: power, W, kW
- $p$: power ratio
- $Q$: volumetric flow rate, m$^3$/s
- $q$: volumetric flow rate ratio

**Greek symbols**

- $\phi$: flow rate number
- $\eta$: efficiency
- $\lambda$: efficiency ratio
- $\pi$: power number
- $\rho$: density, kg/m$^3$
- $\Psi$: head number

**Subscripts**

- $B$: best efficiency point
- max: maximum
- $p$: pump
- $r$: rated point
- $t$: turbine

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![Fig. 1](image_url)
and some ballast loads were selected and installed in the test rig. The positions of barometers were defined individually for each test, depending on pump’s and turbine’s inlet and outlet pipes. Due to slip factor in induction motors, all tested pumps were rotating at 1450 rpm in pump mode. In PAT application, since a synchronous generator was coupled directly, a nominal speed corresponding to one of the synchronous speeds (e.g. 750, 1000, 1500 or 3000 rpm) should be chosen. In this experiment, all PATs were tested at 1500 rpm.

After measuring all parameters, head, flow rate, output power and efficiency of PATs were calculated. A first-order uncertainty analysis was performed using constant odds combination method, based on a 95% confidence level as described by Moffat [15]. The uncertainty of head, flow rate, power and efficiency were ±5.5%, ±3.4%, ±5.1% and ±5.5%, respectively.

4. Results and discussion

Experiments showed that a low-specific-speed centrifugal pump can operate as turbine in different rotational speeds and various heads and flow rates without any mechanical problem. The results are shown in Figs. 3–5. In these figures, \( \psi, \phi, \pi \) are

\[
\psi = \frac{gH}{n^2D^2}, \quad \phi = \frac{Q}{nD^3}, \quad \pi = \frac{P}{\rho n^3 D^5},
\]

where \( H \) (m), \( Q \) (m\(^3\)/s) and \( P \) (W) are head, flow rate and power, respectively. \( n \) (rps) is rotational speed and \( D \) (m).
is impeller’s diameter. Since flow directions in PAT and pump were opposite, the flow rate of PAT is negative in Figs. 3–5.

4.1. Prediction of BEP of a PAT

Figs. 3–5 show that the PAT works in higher flow rate and head in comparison with the pump mode. Fig. 6 shows the BEP of PATs, based on the following dimensionless parameters, which were also used by other researchers [7,10–12]

\[ h = \frac{H_{tb}}{H_{pb}}, \quad q = \frac{Q_{tb}}{Q_{pb}}, \quad p = \frac{P_{tb}}{P_{pb}}, \quad \lambda = \frac{\eta_{t_{\text{max}}}}{\eta_{p_{\text{max}}}}. \] (2)

\( H, \ P, \ Q \) and \( \eta \) are head, power, flow rate and efficiency. Subscripts p, t and b are related to pump, turbine and BEP. Fig. 6 shows that the pumps with higher specific speeds have lower \( h \) and \( q \). The value of \( \lambda \) is almost constant for all pumps with different specific speeds. Variations of \( p \) and specific speed are not proportional.
Using experimental data, some relations were obtained to calculate the BEP of the PAT based on the BEP of the pump mode. These relations are only valid for low-specific-speed centrifugal pumps

\[ \gamma = 0.0233 x_p + 0.6464, \]  
\[ x_t = 0.9413 x_p - 0.6045, \]  
\[ \beta_t = 0.849 \beta_p - 1.2376, \]

where \( x_p (m, m^3/s) \) and \( \beta_p (m, W) \) are the pump and turbine dimensionless specific speeds

\[ x_p = \frac{N_p \cdot Q_{pb}^{0.5}}{(g \cdot H_{pb})^{0.75}}; \]
\[ \beta_t = \frac{N_t \cdot Q_{tb}^{0.5}}{(g \cdot H_{tb})^{0.75}}; \]

Dimensionless parameters \( \gamma, x_t \) and \( \beta_p \) can be defined as

\[ \gamma = \left( h \right)^{-0.5} \frac{N_t}{N_p}, \]
\[ x_t = \frac{N_t \cdot Q_{tb}^{0.5}}{(g \cdot H_{tb})^{0.75}}, \]
\[ \beta_p = \frac{N_p \cdot Q_{pb}^{0.5}}{(g \cdot H_{pb})^{1.25}}. \]

In the above equations, \( H (m) \), \( Q (m^3/s) \), \( P (W) \) and \( N \) (rpm) are head, flow rate, power and rotational speed, respectively. Using Eq. (3) and pump hydraulic characteristics, turbine mode head at BEP was obtained. Eqs. (4) and (5) determine the flow rate and power at BEP, respectively.

Some researchers reported that pumps with nearly the same specific speeds may have very different head ratios \( (h) \) and flow rate ratios \( (q) \). For example, if two pumps have the same specific speeds, the more efficient pump operates as a turbine in greater \( h \) and \( q \). However, between the above-mentioned pumps, the one with bigger impeller’s diameter works more efficiently [2]. Considering experimental data provided by Chapallaz et al. [7], these correlations can be used for pumps with same specific speeds and different impeller’s diameters

\[ h_{new} = h \cdot (0.25/D)^{1/4}, \]
\[ q_{new} = q \cdot (0.25/D)^{1/6}, \]
\[ p_{new} = p \cdot (0.25/D)^{1/10}, \]

where \( D (m) \) is the diameter of impeller. In this study, the impeller’s diameters of all tested pumps were 0.250 m.

Maximum efficiency can be obtained using

\[ \eta_{\text{max}} = \frac{P_{\text{tb}}}{\rho \cdot g \cdot Q_{\text{tb}} \cdot H_{\text{tb}}}. \]

Many methods were developed to predict the BEP of PAT. Table 1 compares the predicted head and flow rate ratios by various methods with experimental data. Figs. 7 and 8 show the deviations of these methods from experimental data vs. the pump specific speeds. Figs. 7 and 8 demonstrate that no single method closely resembles the experimental pattern throughout the whole range of specific speeds. Stepanoff [2] and Sharma [4] methods are only accurate for pumps with specific speeds between 40 and 60 (m,m^3/s). Alatorre-Frank’s method [8] properly predicts the head ratio but estimates higher flow rate ratio than experimental data.

A prediction method is valid only if it accurately predicts both head ratio and flow rate ratio simultaneously for a given range of specific speeds. The suggested method acceptably estimates both head and flow rate ratios for centrifugal pumps with \( N_s < 60 \) (m,m^3/s). Since the presented method was based only on four experimental data, it may not be accurate for all centrifugal pumps. More experimental data can improve its accuracy.

### 4.2. PAT characteristic curves

A PAT may work at off-design conditions, but most prediction methods only predicted the BEP of the PAT. Therefore, estimating the complete characteristic curves of a PAT based on its BEP is very remarkable. Experimental data showed that the dimensionless characteristic curves of all PATs based on their BEP were approximately the same. Mentioned dimensionless head and power curves

<table>
<thead>
<tr>
<th>( N_s ) (m,m^3/s)</th>
<th>( \eta_{\text{max}} ) (%)</th>
<th>( D ) (m)</th>
<th>( h )</th>
<th>( q )</th>
<th>( h )</th>
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<td>1.78</td>
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<td>2.09</td>
<td>2.14</td>
<td>1.48</td>
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<tr>
<td>14.7 [6]</td>
<td>46</td>
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<td>1.63</td>
<td>2.17</td>
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<td>20.7 [11]</td>
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<td>0.160</td>
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<td>1.73</td>
<td>1.84</td>
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<td>1.14</td>
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<td>1.32</td>
<td>1.13</td>
<td>1.38</td>
<td>1.18</td>
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</tbody>
</table>
of a PAT can be estimated as below using second and third order polynomials, respectively

\[
H_t / H_{tb} = 1.0283 \left( \frac{Q_t}{Q_{tb}} \right)^2 - 0.5468 \left( \frac{Q_t}{Q_{tb}} \right) + 0.5314,
\]

\[
P_t / P_{tb} = -0.3092 \left( \frac{Q_t}{Q_{tb}} \right)^3 + 2.1472 \left( \frac{Q_t}{Q_{tb}} \right)^2 - 0.8865 \left( \frac{Q_t}{Q_{tb}} \right) + 0.0452.
\]

Efficiency curve can be obtained for each point using 

\[
\eta_t = \frac{P_t}{(\phi \psi)}.
\]

The curves estimated by this method and experimental data were compared in Figs. 9–11. The results were in good agreement with experimental data. However, it must be noticed that this method can only provide an approximate view of the characteristic curves of PAT. Obviously, more experimental data will improve its accuracy.
4.3. A procedure to select the proper PAT for a small hydro-site

All presented methods only give the turbine mode performance of a pump. But in practice, it is necessary to provide a procedure to select the proper PAT for a specific small hydropower stations. Based on the experimental data, one procedure which uses the rated point can be presented as follows. The rated point is the operating point of the machine, which depends on the site conditions and usually not exactly coincides with design point or the BEP of the machine.

The procedure can be illustrated as below for a small hydro-site:

Step1: The pump specific speed in its rated point, $N_{sp}$ can be calculated as

$$N_{sp} = 0.3705N_{st} + 5.083,$$  \hspace{1cm} (16)

where $N_{st}$ (m, kW) and $N_{sp}$ (m, m$^3$/s) are the turbine and pump specific speeds in their rated points, respectively.

Step 2: $\gamma$ can be obtained by putting $z_p = \frac{N_{sp}}{N_{st}}$ in Eq. (3).

Step 3: Knowing $\gamma$ and using Eq. (8), $h$ is determined.

Step 4: $H_{pr}$ can be calculated by $H_{pr} = \frac{N_{p}}{\beta}$.

Step 5: $Q_{pr}$ can be obtained using $N_{sp}$, choosing $N_{p}$ and knowing $H_{pr}$.

Step 6: The proper PAT can be easily selected when $H_{pr}$, $Q_{pr}$ and $N_{p}$ are known.

This procedure is only valid for turbines with $N_{st} < 150$ (m, kW).

5. Conclusions

In this study, a mini hydropower plant was installed in laboratory as the test rig and four centrifugal pumps ($N_{s} < 60$ (m, m$^3$/s)) were tested as turbines. Experiments
showed that a centrifugal pump can appropriately operate as a turbine in various rotational speeds, heads and flow rates.

A PAT works in higher head and flow rate than those of the pump mode at the same rotational speed. Efficiencies are almost the same in both pump and turbine modes. Pumps with higher specific speeds have lower ratios of $h = \frac{pb}{tb}$ and $q = \frac{pb}{tb}$. Variations of $p = \frac{pb}{tb}$ were not proportional to variations of pump’s specific speed.

A new method was derived to predict the BEP of a PAT based on pump’s hydraulic specifications, especially the specific speed which characterizes the type of the runner and consequently, its hydraulic behavior [18,19]. It was shown that between two pumps with same specific speeds, the more efficient pump operates as turbine in greater $h$ and $q$. On the other hand, the pump with bigger impeller is more efficient in same specific speeds. Some correlations were presented for pumps with same specific speeds and different impeller’s diameters.

The predicted $h$ and $q$ by this method were in good coincidence with the experimental data. This method is only valid for centrifugal pumps with $N_s < 60$ (m, m$^3$/s).

Since a PAT may work at off-design conditions, determining its complete characteristic curves is instrumental. Some relations were presented to estimate PAT complete characteristic curves based on its BEP. Although the results coincide with the experimental data, it must be noticed that this method can only estimate the characteristic curves of PAT.

In practice, selecting the proper PAT for a small hydro-site is in demand. In this study, a procedure was presented to choose a proper centrifugal PAT for a small hydro-site with $N_{st} < 150$ (m, kW).

Future works and more experimental data can improve all suggested methods.

Acknowledgements

The authors gratefully acknowledge the support of IWPC Co. and the contribution of the University of Tehran, Berkeh Pump Co. and PETCO Co.

References