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The comparison of incomplete sensitivities and Genetic algorithms applications in 3D radial turbomachinery blade optimization

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ABSTRACT

In the present work, a centrifugal pump impeller's blades shape was redesigned to reach a higher efficiency in turbine mode using two different optimization algorithms: one is a local method as incomplete sensitivities–gradient based optimization algorithm coupled by 3D Navier–Stokes flow solver, and another is a global method as Genetic algorithms and artificial neural network coupled by 3D Navier–Stokes flow solver. New impeller was manufactured and tested in the test rig. Comparison of the local optimization method results with the global optimization method results showed that the gradient based method has detected the global optimum point. Experimental results confirmed the numerical efficiency improvement in all measured points. This study illustrated that the developed gradient based optimization method is efficient for 3D radial turbomachinery blade optimization.

1. Introduction

The optimization problems associated to turbomachinery design often involve many constraints and large sets of parameters, which in general leads to objective functions presenting many extreme. It is well-known that optimization methods based on gradients techniques are efficient in terms of convergence rate, but do not guarantee to produce the global optimum [1]. On the other hand Genetic algorithms offer the advantage of enhancing the probability of reaching the global optimum, but may require thousands of iterations [2].

The computation of the gradient of the cost function in gradient based methods for optimization is a major problem. Adjoint method based on control theory [3,4] can reduce the cost of this calculation by developing a complicated solver for the adjoint variable. This method is particularly efficient for problems with a large dimension of the control space. This difficulty is more penalizing when industrial black-box flow solvers are used for the state, which is nowadays a systematic demand. Finite differences permit to get the sensitivity of black-box solvers, but then the cost of the evaluation is proportional to the size of the control space. Incomplete sensitivity method is a possible choice for calculating approximate gradient at practically no cost. In addition, incomplete sensitivities can be computed for individual constraint at no extra cost. Providing individual sensitivities for constraints is useful in robust optimization as one would like to see the sensitivity of the final design for small perturbation of control parameters for the different functionals involved. However, incomplete sensitivities have a limited validity domain: the cost function must be defined over the shape, or part of it, and must involve product of state by geometry functions. Functionals based, for instance, on aerodynamic coefficients enter this class. But, that functionals involved in radial blade design do not belong to this validity domain. Derakhshan et al. [5,6] proposed a suitable reformulation of the problem for 3D radial turbomachinery blade optimization.

On the other hand, coupling of Genetic algorithms with a three-dimensional Navier–Stokes solver cannot be considered under the framework of an industrial design process. Demeulenaere and Hirsch [7], presented a methodology that the evaluation of the successive designs was performed using an artificial neural network instead of a flow solver, which permits to use the Genetic algorithms in an efficient way. The accuracy of the optimization depends on the knowledge of the neural network, which is fed by design examples stored in a database. The generality of the formulation of the FINE™/Design3D optimization techniques developed by Numeca [8] allows the objective function to be based on the evaluation of the performance at different working conditions.

The design exercise presented in the paper demonstrates the application of two optimization strategies to the redesign of the impeller blades shape of a centrifugal reverse pump. FINE™/Design3D based on Genetic algorithms and neural network gives

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the global optimum point of the cost function. Therefore, this is an appropriate approach to show the incomplete sensitivities–gradient based method is able to recognize the global optimal points.

After numerical redesign, the new impeller was manufactured and tested in the test rig.

2. Efficiency improvement of centrifugal reverse pump

The main objective was to reach higher efficiency by redesigning of the blades. Using the gradient based optimization algorithm and incomplete sensitivity method developed by Derakhshan et al. [5,6] coupled by FINE™/Turbo as the flow solver developed by Numeca [8], the shape of blades was redesigned. In the next step, FINE™/Design3D based on Genetic algorithms and artificial neural network optimization method developed by Numeca [8] applied to redesign the blades again.

2.1. Incomplete sensitivities–gradient based optimization algorithm

The general form of a shape optimization problem can be written as [9]:

$$\begin{cases} \min J(x_c), W(x_c), \nabla_{x_c} W(x_c)) \\ S(x_c, W(x_c), \nabla_{x_c} W(x_c)) = 0 \\ g_1(x_c) \leqslant 0, g_2(W(x_c)) \leqslant 0 \end{cases} \tag{1}$$

where x_c is the control variable for the shape, W is the flow variable, S is the state equation, g_1 is the geometrical constraints, g_2 is the state constraints and J is the cost function that should be minimized.

The local optimization algorithm can be summarized as:
Optimization loop

- Provide initial shape parameterization, x_c^1.
- For k = 2,3 ,..., k_{max} Do:
- Compute the flow state: $W(x_c^k)$.
- Compute the cost function: $J(x_c^k, W(x_c^k))$.
- Compute the incomplete sensitivity of the cost function: $\frac{dJ(x_c^k, W(x_c^k))}{dx_c}$.
- If $\left(\left|\frac{dJ(x_c^k, W(x_c^k))}{dx_c}\right| < \varepsilon \text{ or } J(x_c^k, W(x_c^k)) < \varepsilon\right)$: STOP.
- Compute x_c^{k+1} minimizing J using incomplete gradient and the approximate inverse of Hessian by Broyden, Fletcher, Goldfarb, and Shanno (BFGS) [10] and evaluating when it is necessary $W(x_c^{k+1})$ and $J(x_c^k, W(x_c^k))$.

2.1.1. Shape parameterization

Several parameterizations are possible to describe aerodynamic or hydrodynamic shapes. In radial turbomachinery, one can be considered as the spanwise blade angle distribution from leading to trailing edges. The performance of a radial turbomachines (i.e. centrifugal pumps) is intensely influenced by these blade angles [11]. Previous results have shown that hydraulic efficiency is not sensitive to small perturbations of blade thicknesses [11]. On the other hand, thickness is one of the manufacturing constraints. Therefore the blade thicknesses were frozen in optimization process. Also in this optimization, other manufacturing constraints were ignored. The meridional plan of the hub and shroud and the outlet diameter (in centrifugal pump) were fixed. The optimization was performed in two steps:

2.1.1.1. Primal optimization. For a radial blade, the camber lines in hub-span, mid-span and shroud-span were linked through the following relation:

$$d\theta(R) = \theta(R) - \theta_{initial}(R) = \frac{c_1(R-R_1) + c_2(R-R_1)^2 + c_3(R-R_1)^3}{R} \tag{2}$$

where θ is the tangential angle, R the blade radius, R_1 the radius of the blade leading edge in hub-span (in centrifugal pump). The profile is fixed in R_1. The coefficients c_1, c_2 and c_3 are control parameters (Fig. 1a).

2.1.1.2. Final optimization. The optimal shape from primal parameterization was used as initial guess for the second level parameterization. The camber lines of mid-span and shroud-span were linked to the blade camber line in hub-span through:

$$d\varphi = \frac{c_{2i-1}S + c_{2i-2}S^2 + c_{2i-3}S^3}{S} \tag{3}$$

where i = 2,3. $m = \int \frac{dm}{R}$, $dm = \sqrt{dR^2 + dz^2}$, in $m - \theta$ conformal plan. $S = \sqrt{m^2 + \theta^2}$. Here the blade camber lines in mid-span and shroud-span are rotated around trailing edge, which is fixed with respect to the hub-span camber line of the blade (Fig. 1b). The coefficients c_4, c_5 and c_6 are control parameters for mid-span and c_7, c_8 and c_9 for shroud-span.

2.1.2. Sensitivity and incomplete sensitivity

The gradient of a cost function $J(x_c, q(x_c), W(q(x_c)))$, function of shape control parameters x_c, geometric entities $q(x_c)$ (normal, volume, surface, ...) and state variables W can be derived using chain rule:

$$\frac{dJ}{dx_c} = \frac{\partial J}{\partial x_c} + \frac{\partial J}{\partial q}\frac{\partial q}{\partial x_c} + \frac{\partial J}{\partial W}\frac{\partial W}{\partial q}\frac{\partial q}{\partial x_c} \tag{4}$$

where f and g are functions involving geometric quantities and state quantities respectively. For incomplete sensitivities application, the cost function should be expressed as a function of the aerodynamic coefficients or more generally [3]:

$$J = \int_\Gamma f(x, q(x)) g(W(q(x)) d\gamma \tag{5}$$

The dominant part in the gradient comes from geometrical quantities sensitivities and not from state linearization [3]. More precisely, the last term in gradient expression can be neglected:

$$\frac{dJ}{dx_c} \approx \frac{\partial J(W)}{\partial x_c} + \frac{\partial J(W)}{\partial x_q}\frac{\partial x_q}{\partial x_c} \tag{6}$$

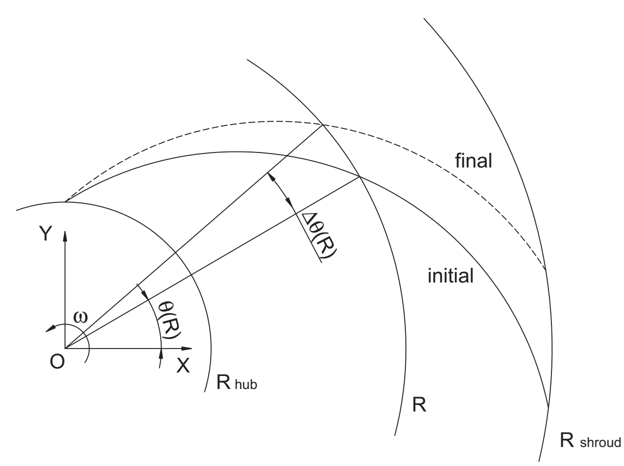

Fig. 1. Radial blade parameterization. (a) First parameterization, blade angle distribution from leading to trailing edges. (b) Second parameterization, rotation mid-span and shroud-span around leading edge with respect to hub-span.

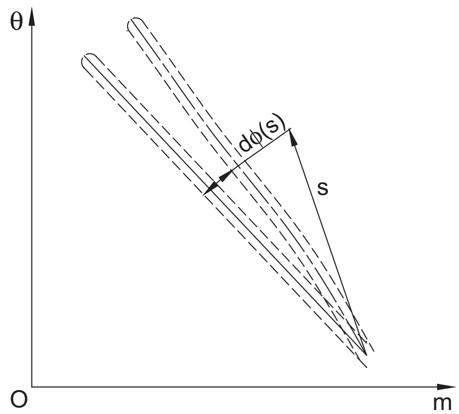

Fig. 1 (*continued*)

This gradient approximation avoids the evaluation of an adjoint state and decreases the computational cost. Typical functionals in this class are aerodynamic or hydrodynamic forces on a shape along an arbitrary direction as:

$$T_r = \left(\int_\Gamma [T \cdot n] d\Gamma \right) \cdot \sigma \tag{7}$$

where $T = -pI + (\nu + \nu_t)(\nabla u_t + \nabla u^T)$. Here, n is the normal to the shape, σ is an arbitrary direction and T is the Newtonian stress tensor.

In this optimization, incomplete sensitivities were improved by adding supplementary terms to add physical sense to the approximate gradient. In other word, reduced order models (i.e. wall functions) can improve incomplete sensitivity in an inexpensive way. The method and its formulation can be found in authors pervious works [5,6].

2.1.3. Cost function for incomplete sensitivities domain

The aim of this study was blade shape optimization of pump impeller in reverse operation to reach higher efficiency in its rated point defined as:

$$\eta_h = \frac{T_r \omega}{\gamma q h} \tag{8}$$

where h is head (m), q is flow rate (m^3/s), T_r (N m) is the axial torque from fluid to impeller, $\omega = \frac{2\pi N}{60}$ and γ is specific gravity (kg/m^2 s^2).

To use incomplete sensitivities, the cost function must be based on information over the shape (or part of it). In Eq. (8) increasing the torque improves the efficiency: $J = -\frac{T_r}{T_{r0}}$ where:

$$T_r = \int_{\Gamma_w} [T \cdot n] R d\Gamma, T = -pI + (\nu + \nu_t)(\nabla u_t + \nabla u^T)$$

But one may need to improve the hydraulic efficiency of the pump at constant design point (constant specific speed). So looking for higher hydraulic efficiency should be done at given head (total pressure difference between outlet and inlet) and flow rate. The flow rate is constant in optimization process and can be imposed through boundary conditions. Head (or pressure difference) can be added as a penalty in the cost function:

$$J = -\frac{T_r}{T_{r0}} + \alpha \frac{|h - h_0|}{h_0} \tag{9}$$

Unfortunately, this new term does not enter to incomplete sensitivity validity domain as it is defined away from the shape and also does not include any geometric quantity. Eventually, the cost function accounting for constant head can be reformulated to adapt incomplete sensitivity method using reformulated pressure difference (or head) based on axial and radial forces and blade volume in radial turbomachinery:

$$J = -\frac{T_r}{T_{r0}} + \alpha \frac{|F_a - F_{a0}|}{F_{a0}} + \beta \frac{|F_R - F_{R0}|}{F_{R0}} + \gamma \frac{|V_b - V_{b0}|}{V_{b0}} \tag{10}$$

which enters incomplete sensitivity validity domain. Indeed, the cost function is the rotor axial torque with state constraints on hydrodynamic axial (F_a), radial forces (F_r) and geometrical constraint on the blade volume (V_b). The details of this reformulation can be found in authors' previous work [5]. Hydraulic efficiency can be improved by increasing torque. Head can be unchanged by keeping F_a, F_R and V_b at the same time.

2.1.4. Black-box sensitivity evaluation

Obviously, incomplete sensitivities can be obtained by linearizing the functional and keeping all state based quantities unchanged. However, it might be interesting to avoid any extra programming effort for the user. This is a demand from industry where people are often not professional enough or are black-box solver users. This is one of main interests of gradient free approaches such as Genetic algorithms. A possible implementation of incomplete sensitivities is a change in the functional does not imply any new coding for the calculation of the gradient other than coding the functional itself.

2.1.5. Complex variable method

The drawbacks of difference formula are well known (choice of the increment and difference between two close quantities). These can be avoided working with complex values. In practice, this method only requires a redefinition of all real variables of a computer program as complex. This is not convenient if a black-box solver is used. But, with incomplete sensitivities only the boundary integral calculation are involved [5,6].

2.1.6. Minimization method

We briefly recall the minimization method to show where incomplete sensitivities appear in descent iterations. Our main interest goes to quasi-Newton methods such as BFGS coupled with a line search method [10]. The approximate inverse of Hessian of the functional is built using successive gradient evaluations. Therefore, with incomplete sensitivities one might expect not only a deviation in the gradient but also in eigenvalues of the Hessian. But it has been showed that for functionals in the validity domain of incomplete sensitivities one has equidistribution of the error in the Hessian up-to-date. [5,6].

2.2. Genetic algorithms and artificial neural network optimization method

FINE™/Design3D developed by Numeca [8] is an optimal design software with various optimization methods. One global method is Genetic algorithms coupled by artificial neural network. The basic idea of this method is to accelerate the design of new blades using the knowledge acquired during previous designs of similar blades. The core of the design system is a database containing the results of all Navier–Stokes computations performed during the previous and present design processes.

2.2.1. The parametric blade modeler (AutoBlade™)

The parametric model that has been adopted in Autoblade™ consists of three sections at hub-span, mid-span and shroud-span, defined by a camber line (Fig. 2) and symmetric thickness distributions. The trailing edge radius is constant from hub to shroud, whereas the meridional trace of the leading edge is non-linear. 3D blade shape is stacked along the trailing edge, with a non-linear

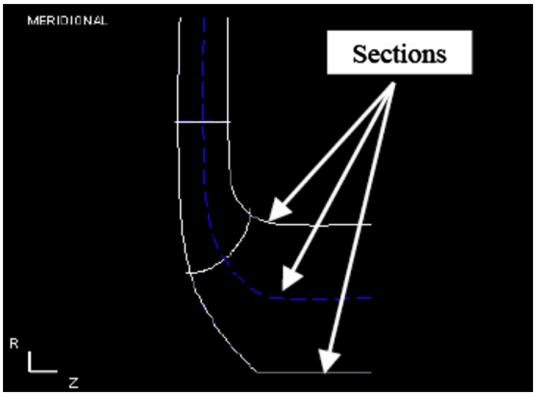

Fig. 2. Meridional view of the centrifugal pump.

tangential stacking model based on the two angles at hub and shroud.

The number of parameters included in the optimization has been restricted to 12. The hub and shroud endwalls, as well as the thickness distributions have been kept constant. The camber line parameters are: four control points on each section. As indicated in Fig. 3, the 4th control point has been eliminated from the optimization. It has been set as a dependent parameter, whose value results from the values of the 3rd and 5th control points positions.

2.2.2. The objective function

The optimization problem can therefore be seen as the minimization of an objective function in function of several variables (the geometrical parameters) subject to several constraints (mechanical, manufacturing and aerodynamic constraints), the main objective function, and the constraints being non-linear. The general approach to this problem is to transform the original constrained minimization problem into an unconstrained one by converting the constraints into penalty terms that are increasing when violating the constraints. A pseudo-objective function is then created by summing up all the penalty terms and the original objective:

$$F = \sum_{penalties} w\left(\frac{V - V_{req}}{V_{req}}\right)^2 \tag{11}$$

where w is a weighting function that the user can associate to each penalty. One can notice that the difference between the actual value V and the required V_{req} is divided by a reference value V_{ref}, so that all the terms have a similar order of magnitude.

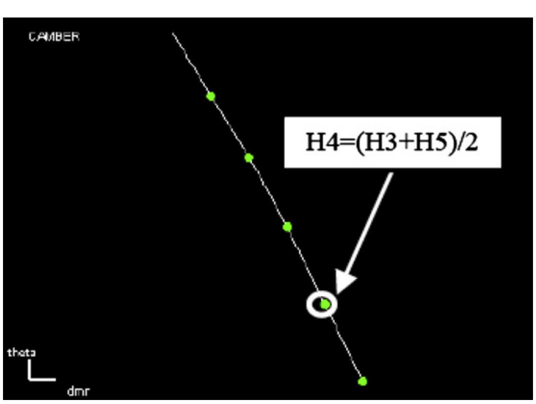

Fig. 3. Setting of dependent parameter.

The optimization technique that is adopted in FINE™/Design3D can be considered as multi-objective, as all objectives and constraints are put together into one single objective function. There is no guarantee that the final proposed solution satisfies all the constraints. Weighting functions are associated to the different constraints, which allow the user to reflect the levels of priority into the optimization. Different solutions will be obtained, depending on the values of the weighting factors. Here objective function was included efficiency and total pressure difference with following values: $\eta_{req} = \eta_{ref} = 1.0$ and $\Delta p_{ref} = \Delta p_{req} = \Delta p_0$. The algorithm tries to increase efficiency around initial total pressure difference.

The objective function was the same with GB optimization method.

2.2.3. The optimization algorithm

The goal of the optimization is to find the minimum of the objective function using the simplified analysis model. Here, an essential issue is the robustness of the numerical optimization algorithm. The choice of the optimization algorithm is mainly based on the following two considerations

- Many local optima may exist in the design space and therefore a global optimization technique is required.
- The evaluation of the blade performance using the approximate model is very fast.

Consequently, the number of required function evaluations is now of far less importance than if a detailed Navier–Stokes computation was needed at each step.

Based on the first consideration, the straightforward application of numerical optimization techniques that rely on derivatives are questionable because they are only local optimization techniques. On the contrary, stochastic techniques such as the Genetic algorithm (GA) or simulated annealing (SA) are global optimization techniques that do not get stuck in local minimum and therefore offer an alternative to conventional gradient methods for optimization problems where the function evaluation is very fast.

Genetic algorithms were designed by Holland in the 70 s, and improved and made well known by Goldberg [2]. The Genetic algorithm is initiated with the creation of an initial population whose elements are randomly selected in the whole design space. Different procedures are then applied in order to successively generate new populations containing better elements. The performance of an individual is measured by its fitness (inverse of the objective function). Elitism can first be applied, which consists of keeping the best elements from the current population. The other applied operations are combination and mutation. Pairs of individuals are selected from this population based on their objective function values.

Then, each pair of individuals undergoes a reproduction mechanism to generate a new population in such a way that fitter individuals will spread their genes with higher probability.

The children replace their parents. As this proceeds, inferior traits in the pool die out due to lack of reproduction. At the same time, strong traits tend to combine with other strong traits to produce children who perform better.

2.2.4. The approximate model

The basic principle of the method is to build an approximate model of the original analysis problem (the three-dimensional Navier–Stokes equations). This approximate model can then be used inside an optimization loop instead of the original model.

In this way the performance evaluation by the approximate model is not costly and the number of performance evaluations performed by the approximate model for the optimization is no longer critical.

Among the large number of possible techniques able to construct the approximate model, artificial neural network (ANN) has been selected. Although the initial motivation for developing ANN was to develop computer models that could imitate certain brain functions, ANN can also be thought of as a powerful interpolator.

The details can be found in [12].

2.3. 3D flow simulation

To have an efficient shape optimization for fluids, the optimization platform should be able to interact with various CFD solvers. To achieve such adaptability, it is important to keep the interface free of constraints for a particular software. This is also one of the advantages of incomplete sensitivities concept as it lets the interface to be only surface based. FINE™/Turbo developed by Numeca, is an integrated software based on finite volume discretization for multi-block structured grids. The multi-block structured grids on the blades were prepared by AutoGrid5™ developed by Numeca [13]. The physical model used in the solver was the Reynolds-averaged Navier–Stokes equations in rotating frames of reference coupled with various turbulence models and near-wall treatment for low-Reynolds modeling. The standard high Reynolds $k-\varepsilon$ turbulence model with extended wall functions could be chosen without any limitation [14–16].

The discrete schemes were second order in space [14] and first-order in time with time marching to steady solutions. Mass flow rate, velocity direction, turbulence kinetic energy k and turbulent dissipation ε were imposed at inlet boundary while at outlet boundary condition, static pressure was prescribed. Finally, periodic boundary condition was applied between two blades. This flow solver is used for 3D flow simulation in both optimization methods.

3. Experimental setup

A complete laboratory model of mini hydropower plant was installed in University of Tehran [17] as shown in Fig. 4. The flow rate and head for pump working as turbine were generated in the experimental setup by several pumps.

When a pump works as a turbine, a control system is needed to automatically regulate the frequency. The classical governor used for standard turbines are expensive and not always recommended for small hydropower plants. Since these types of plants are more being used in isolated areas, an electronic load controller with ballast loads was built and used for keeping the frequency of generator in these tests. A conventional synchronous generator was installed for producing electricity. For turbine shaft torque measuring, generator was changed to suspense state mode and using a scaled arm and several weights, turbine shaft torque was measured. The flow rate was measured by the discharge law and using various orifice plates for each test. Pressures were measured by some barometers. An industrial low specific speed centrifugal pump with specific speed 23.5 (m, m^3/s) was selected for testing as turbine with one original impeller and three modified impellers. This pump had maximum input turbine shaft power, maximum head and maximum flow rate of 20 kW, 25 m and 120 l/s, respectively. For the reverse pump testing, feed pump, several pipes, an orifice, a generator and ballast loads were selected and installed in the test rig. In the application of the reverse pump, it should be considered that: if a generator is to be coupled directly, a nominal speed corresponding to one of synchronous speeds (e.g. 750, 1000, 1500 or 3000 rpm) should be chosen. For induction generators and also induction motors slip factor must be taken into account (the tested pump rotates at 1450 rpm in pump mode). In practice, synchronous generators are usually used. The reverse pump was tested in N_t = 1500 rpm.

After measuring all parameters, reverse pump head, flow rate, output power and efficiency were obtained. A first-order uncertainty analysis is performed using constant odds combination method, based on a 95% confidence level as described by Moffat [18]. The uncertainty of head, flow rate, power and efficiency are, respectively ±5.5%, ±3.4%, ±5.1% and ±5.5%.

4. Results

In this study a centrifugal pump considered in reverse rotation with a rotational speed of, 1500 rpm, a flow rate of 126 m^3/h and a total head rise of, 38 m. The pump had seven blades with an inlet radius in hub of 0.25 m. This pump was tested as turbine in the test rig. The shape of impeller blades were optimized by developed incomplete sensitivities–gradient based optimization algorithm to reach higher efficiency in rated point region. The cost function for optimization was:

$$J = -\frac{T_r}{T_{r0}} + 0.05\frac{|F_a - F_{a0}|}{F_{a0}} + 0.1\frac{|F_R - F_{R0}|}{F_{R0}} + 0.001\frac{|V_b - V_{b0}|}{V_{b0}}$$

The initial geometry was available at hub-span, mid-span and shroud-span. The mesh used for FINE™/Turbo was structured and multi-block and of HHOHH(O5H)-type. This was an elliptic mesh with about 400,000 nodes. Computational domain and grid view in mid-span are shown in Fig. 5. To check if the grid was too coarse, simulations were made with one impeller channel and two different grids. The first grid had about 150,000 cells for one impeller channel and the second one consisted of about 400,000 cells. The simulation results showed differences of less than 1% for efficiency and head. The cost function included not only the torque to maximize but also a state constraint on hydrodynamic forces as well as a geometrical constraint on the blade volume.

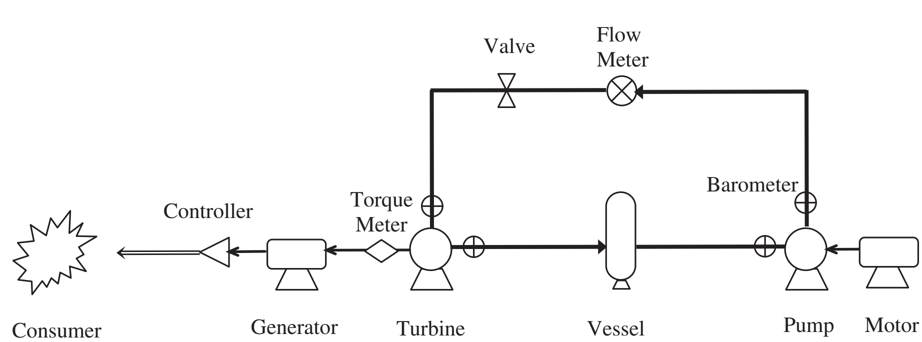

Fig. 4. The mini hydropower established in University of Tehran.

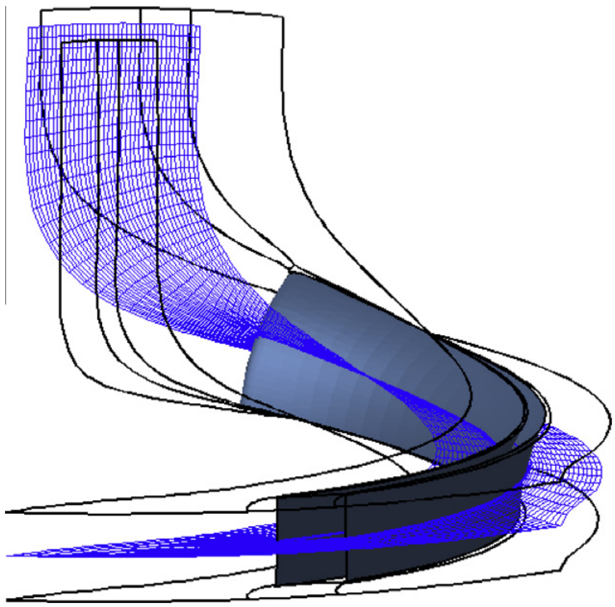

Fig. 5. Computational domain and the grid on mid-span.

Results showed that the torque was increased by 4.25% and the head by 1.97% for a hydraulic efficiency improvement of 2.2% (Fig. 6 and Table 1).

For the second optimization with the same cost function, the torque was increased by 2.27%, and the head by 1.08% for a hydraulic efficiency improvement of 1.17% (Fig. 7 and Table 1).

The initial and final optimization results are shown in Table 1. The final designs were more robust than the original shape as the gradients of all constraints were reduced. The optimization process was reasonably fast and required about 17 iterations of the optimization algorithm and 26 functional evolutions. On a 3 GHz computer with 4GB RAM, the flow analysis and the complete optimization took almost 23 h and 25 h, respectively.

In the next step, the impeller of reverse pump redesigned by FINE™/Design3D with described parameterization and objective function with the same simulation of gradient based method. For training of ANN, 10 data bases evaluated using flow solver. After training, optimization loop converged after 15 iterations. Fig. 8 shows the history of objective function evaluation during optimization loop. Fig. 9 compares the optimization results of Genetic

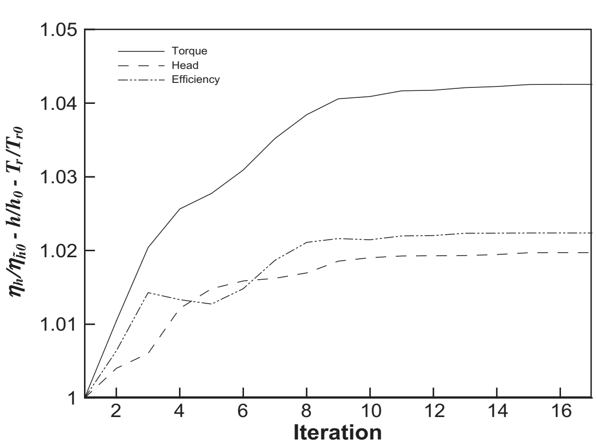

Fig. 6. GB-optimization for the first parameterization of a centrifugal reverse pump blade. (a) Blade performance vs. optimization iterations (η_h/η_{h0}, h/h_0, T_r/T_{r0}). (b) Initial and final blades, hub-span, mid-span and shroud-span.

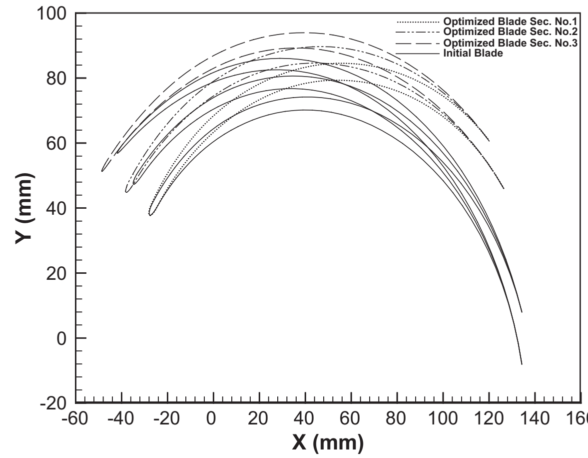

Fig. 6 (*continued*)

Table 1
Optimization results for the gradient based method.

	Initial	Primal optimization	Final optimization
T_r/T_{r0}	1.0	1.0425(+4.25%)	1.0227(+2.27%)
h/h_0	1.0	1.0197(+1.97%)	1.0108(+1.08%)
η/η_0	1.0	1.0223(+2.23%)	1.0117(+1.17%)
Cost function gradient	0.045	0.006	0.004

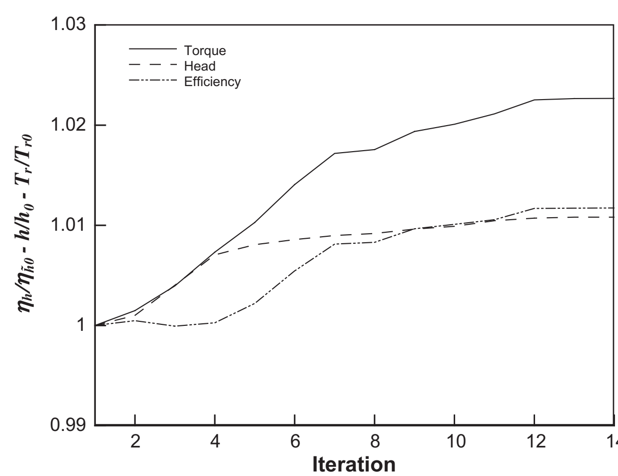

Fig. 7. GB-optimization for the second parameterization of a centrifugal reverse pump blade. (a) Blade performance vs. optimization iterations (η_h/η_{h0}, h/h_0, T_r/T_{r0}). (b) Initial and final blades, hub-span, mid-span and shroud-span.

algorithms with the initial geometry results. Genetic algorithms results showed that the torque was increased by 5.37% and the head by 2.2% for a hydraulic efficiency improvement of 3.1% (Table 2). Fig. 10 compares the results of gradient based method (GB) and Genetic algorithms (GA) results. It observes that GB has detected global minimum point properly. On a 3 GHz computer with 4GB RAM, complete GA optimization took almost 30 h.

Finally, the new impeller were manufactured and tested in the test rig.

Figs. 11 and 12 show the results of the experiments. In these figures, ψ, ϕ, π are defined as:

$$\psi = \frac{gH}{n^2D^2}, \quad \phi = \frac{Q}{nD^3}, \quad \pi = \frac{P}{\rho n^3 D^5} \tag{12}$$

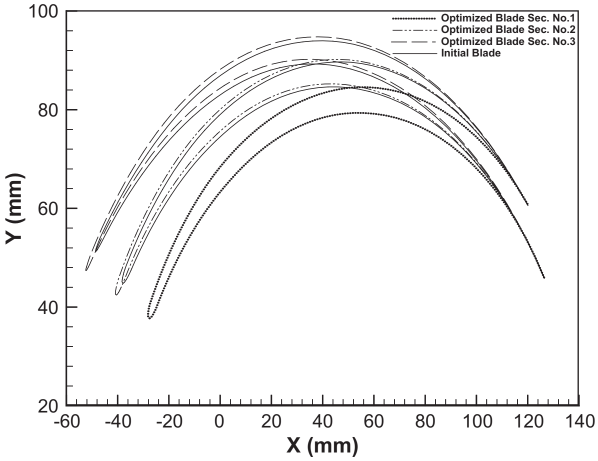

Fig. 7 (*continued*)

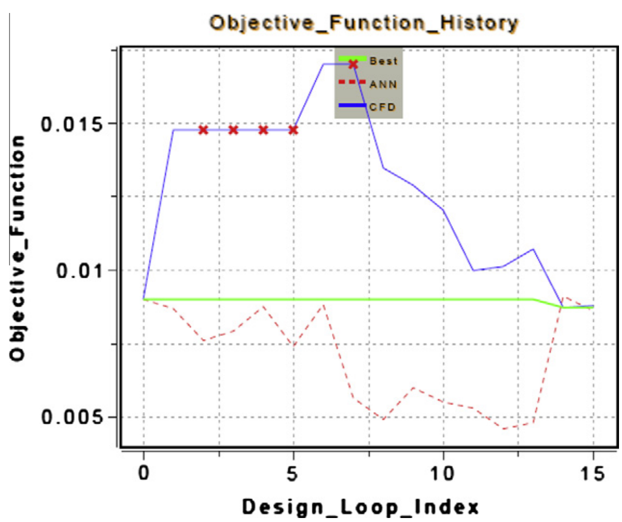

Fig. 8. Evaluating of objective function during GA optimization.

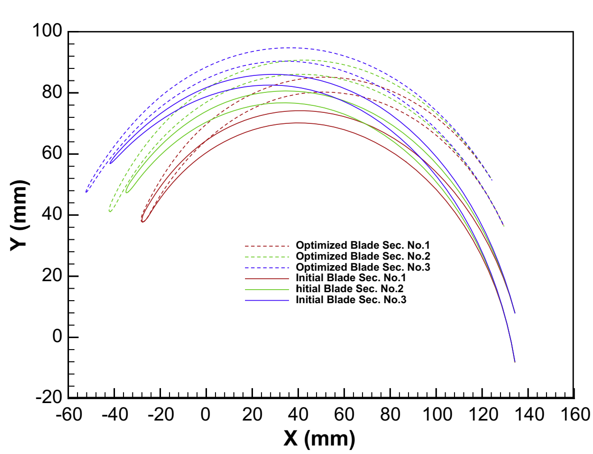

Fig. 9. Initial and final blades for GA, hub-span, mid-span and shroud-span.

Table 3 shows the changes in hydraulic parameters in flow rate of BEP. Efficiency improvement occurs in all flow rates of part load and overload zones. Optimized impeller gives rising as −2.2%, +9.4%, +14.8% and +2.9% for head, power and efficiency, respectively. Table 4 shows the comparison between experimental and

Table 2
Optimization results for the Genetic algorithms method.

	Initial geometry	Final geometry
T_r/T_{r0}	1.0	1.0537(+5.37%)
H/H_0	1.0	1.022(+2.2%)
η/η_0	1.0	1.031(+3.1%)

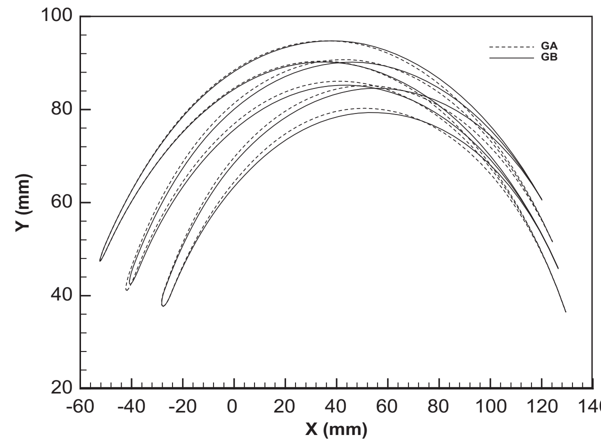

Fig. 10. Comparison of the final blade designs of GB and GA.

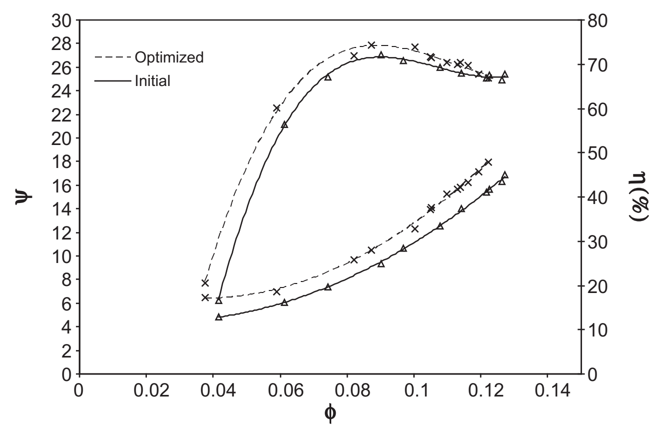

Fig. 11. Experimental results for head number and efficiency.

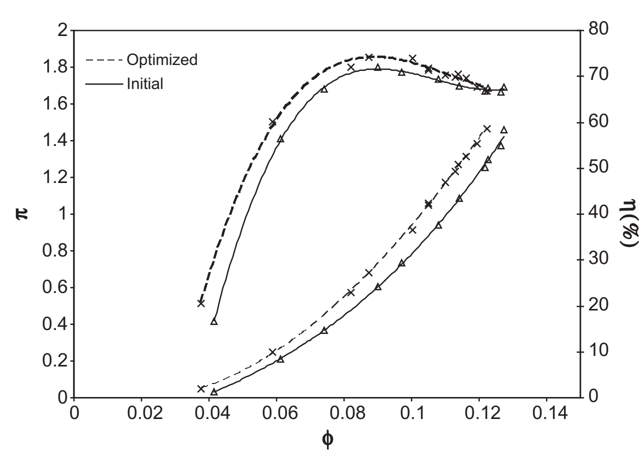

Fig. 12. Experimental results for power number and efficiency.

Table 3
Experimental results for new implore.

	Initial geometry	Optimized geometry
φ	0.090	0.088(−2.2%)
ψ	9.6	10.5(9.4%)
p	0.61	0.70(14.8%)
η	0.725	0.746(2.9%)

Table 4
Comparison of numerical optimization and experimental results.

	Initial	Numerical optimization	Experimental
T_r/T_{r0}	1.0	+9.65	+14.8
h/h_0	1.0	+1.82	+9.4
η/η_0	1.0	+2.61	+2.9

numerical optimization results. Experiment shows higher values for power, head and efficiency. This confirms the numerical results and shows that the blade behavior was improved for a wider operating range. But the head is increased slightly more than numerical optimization data. In the optimized geometry, the inlet blade angle is bigger than that of the initial one.

5. Conclusions

Using the gradient based optimization process on the radial turbomachinery blade design developed in authors' previous works [5,6], the blades shape of the impeller of a reverse pump was optimized to improve its maximum efficiency in rated point. The new design was numerically compared by the global optimization results – Genetic algorithms and neural network-of FINE™/Design3D and experimentally tested for several operating points. It is observed that the gradient based optimization method has detected the global minimum point properly. Experimental results confirmed that the efficiency is improved in all measured point for new impeller.

The main aim was to prove that incomplete sensitivity could be given confidence using GA as a reference solution provider. But it is well-known that the gradient based methods, in general, converge to local optimums and there is no guarantee that they find the global optimum.

This study illustrated that developed gradient based optimization method reached to the global optimization method faster than conventional global optimization methods for this special case as the turbomachinery blade optimal design (not in general).

References

[1] Vanderplaats GN. Numerical optimization techniques for engineering design. New York: McGraw-Hill; 1984.
[2] Goldberg DE. Genetic algorithms. Addison Wesley; 1994.
[3] Mohammadi B, Pironneau O. Applied shape optimization for fluids. 2nd ed. Oxford University Press; 2009.
[4] Mohammadi B, Pironneau O. Shape optimization in fluid mechanics. Annu Rev Fluid Mech 2004;36:255–79.
[5] Derakhshan S, Mohammadi B, Nourbakhsh A. Incomplete sensitivities for 3D radial turbomachinery blade optimization. Comput Fluids 2008;37:1354–63.
[6] Derakhshan S, Mohammadi B, Nourbakhsh A. Efficiency improvement of centrifugal reverse pumps. ASME J Fluids Eng 2009;131:021103-1-9.
[7] Demeulenaere A, Hirsch Ch. Application of multipoint optimization to the design of turbomachinery blades. ASME Paper GT 2004:53110.
[8] FINE™/Design3D user guide. Numeca; 2007.
[9] Pironneau O. Optimal shape design for elliptic systems. New York: Springer; 1984.
[10] Nocedal J, Stephen JW. Numerical optimization. New York: Springer; 2006.
[11] Nourbakhsh A, Jaumotte A, Hirsch C, Parizi H. Turbopumps and pumping systems. GmbH: Springer-Verlag; 2007.
[12] Demeulenaere A, Lighout A, Dijkers R, Purwanto A, Hirsch Ch, Visser F. Design and optimization of an industrial pump: application of Genetic algorithms and neural network. ASME Fluids Engineering Division Summer Meeting & Exhibition; 2005. p. 1–9.
[13] Fine™/Turbo V.8 user guide. Numeca; 2007.
[14] AutoGrid™ V.5 user guide. Numeca; 2007.
[15] Hakimi N. Preconditioning methods for time dependent Navier–Stokes equations. PhD thesis, Vrije Universiteit Brussel; 1997.
[16] Launder B, Spalding D. The numerical computation of turbulent flow. Comput Methods Appl Mech 1974;3:269–89.
[17] Derakhshan S, Nourbakhsh A. Experimental study of characteristic curves of centrifugal pumps working as turbines in different specific speeds. Exp Therm Fluid Sci 2008;32:800–7.
[18] Moffat RJ. Contributions to the theory of single-sample uncertainty analysis. ASME J Fluids Eng 1982;104:250–60.