Damage Mechanics in Composite Materials

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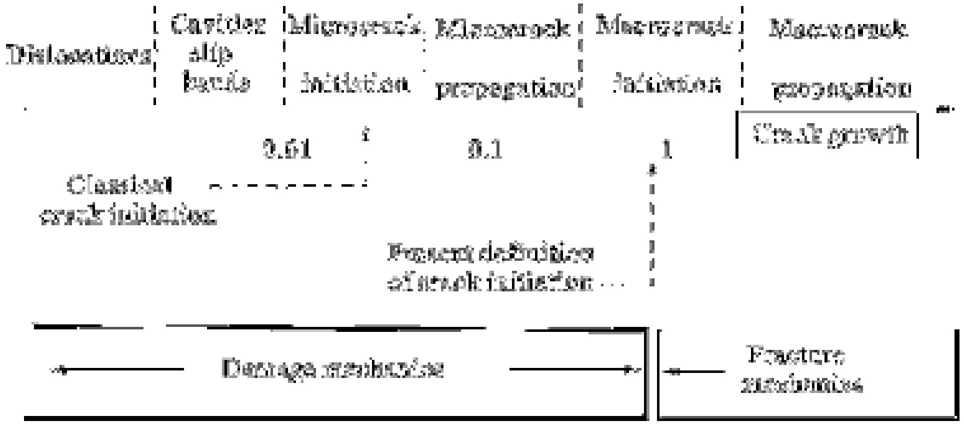
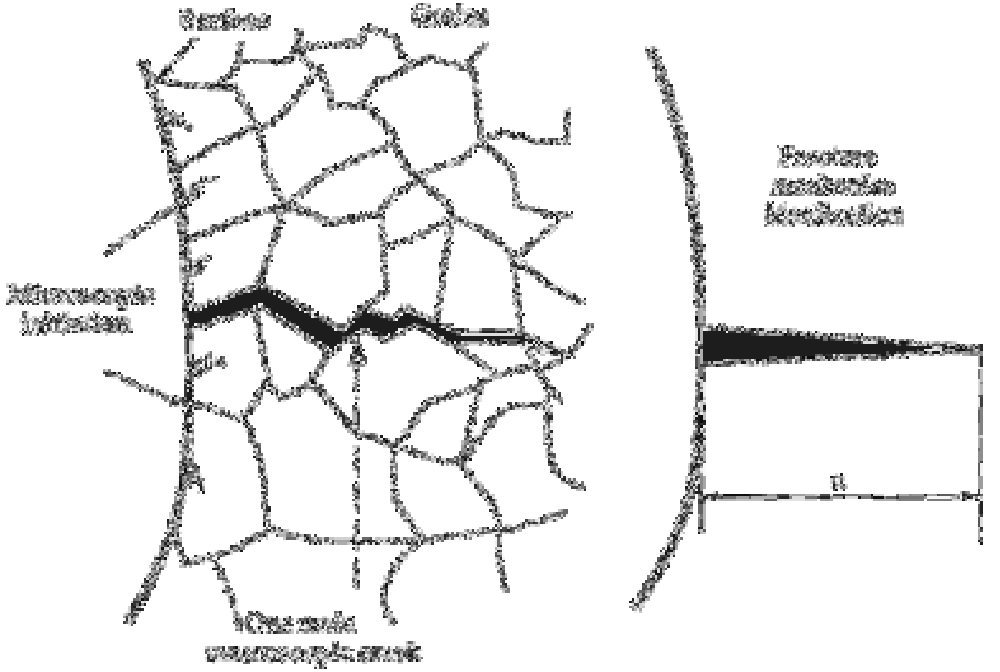
Introduction

- ***CDM has been widely used in the modeling of the progression of distributed microscopic damage in composite laminates.***

- ***In CDM, Damage variables can be presented through <u>the internal state variables</u> of thermodynamics for irreversible processes.***

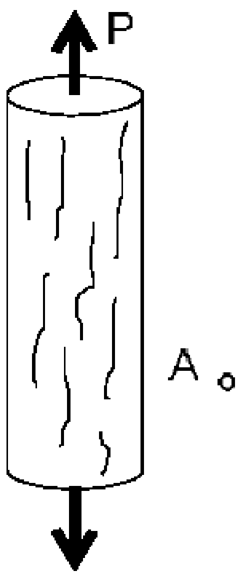
Macroscopic crack initiation



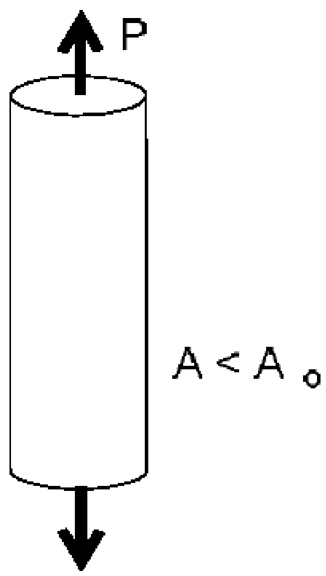


Concept of Continuum Damage Mechanics

- In the continuum damage mechanics, a damage state is replaced by a fictitious un-damaged state with reduced properties.



$$\sigma = \frac{P}{A_o}$$



$$\bar{\sigma} = \frac{P}{A}$$

$$\sigma = \frac{P}{A_o}$$ In damage state

$$\bar{\sigma} = \frac{P}{A}$$ In un-damage state

$$\varphi = \frac{A_o - A}{A_o}$$ Defining φ as a Damage parameter

$$\bar{\sigma} = \frac{\sigma}{1-\varphi} = M(\varphi)\sigma$$

Stress transformation between effective fictitious undamaged and damaged states in multi-axial loading

$$\overline{\sigma} = M(\varphi) : \sigma$$

which M is a forth order damage tensor as follows in principle axes of loading:

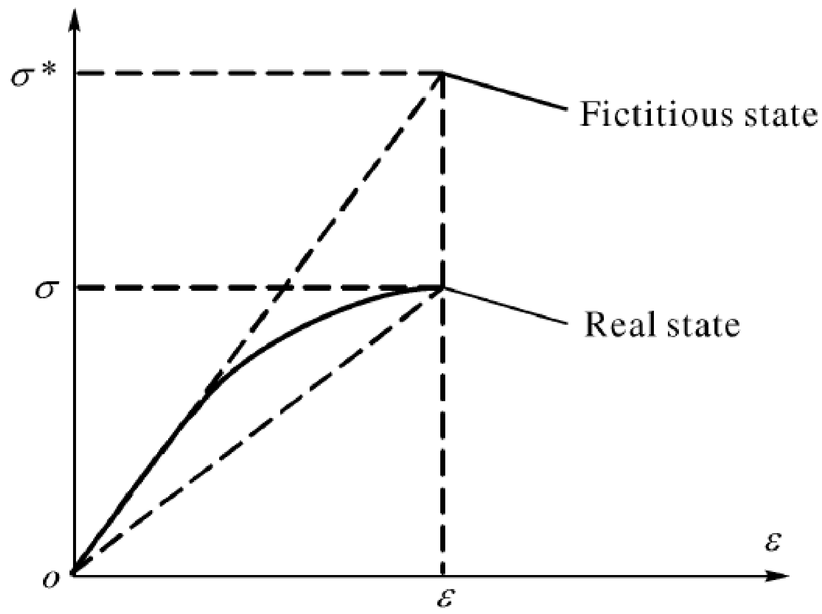
$$M_{ijkl} = \begin{bmatrix} \frac{1}{1-\varphi_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1-\varphi_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1-\varphi_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{1-\varphi_2}\sqrt{1-\varphi_3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{1-\varphi_1}\sqrt{1-\varphi_3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{1-\varphi_1}\sqrt{1-\varphi_2}} \end{bmatrix}$$

Different Basic Hypothesis of Damage Mechanics

- ***Hypothesis of Strain Equivalence***
- ***Hypothesis of Stress Equivalence***
- ***Hypothesis of Elastic Energy Equivalence***
  - ***Hypothesis of Elastic Strain Energy Equivalence***
  - ***Hypothesis of Complementary Energy Equivalence***

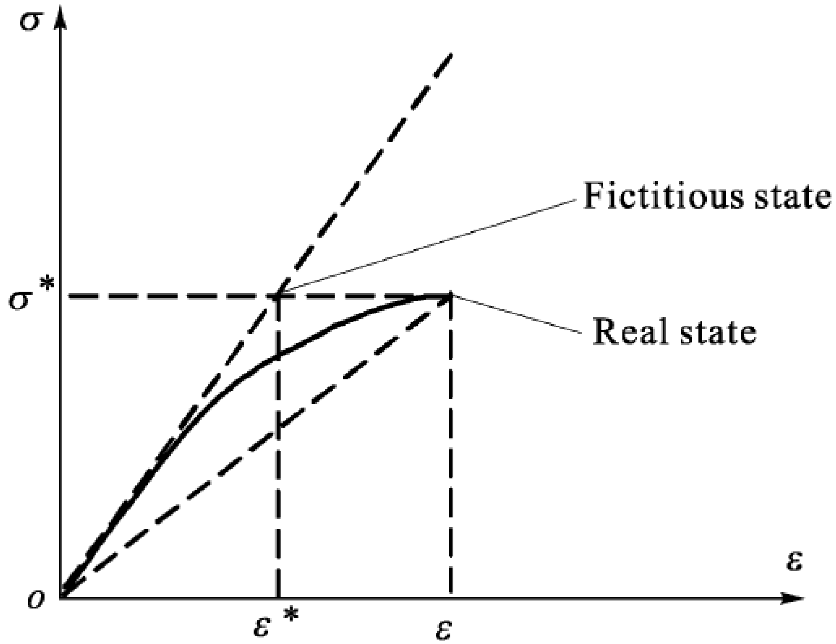
Hypothesis of Strain Equivalence

- *The strain associated with a damaged state under the applied stress is equivalent to the strain associated with its fictitious undamaged state under the effective stress.*



Hypothesis of Stress Equivalence

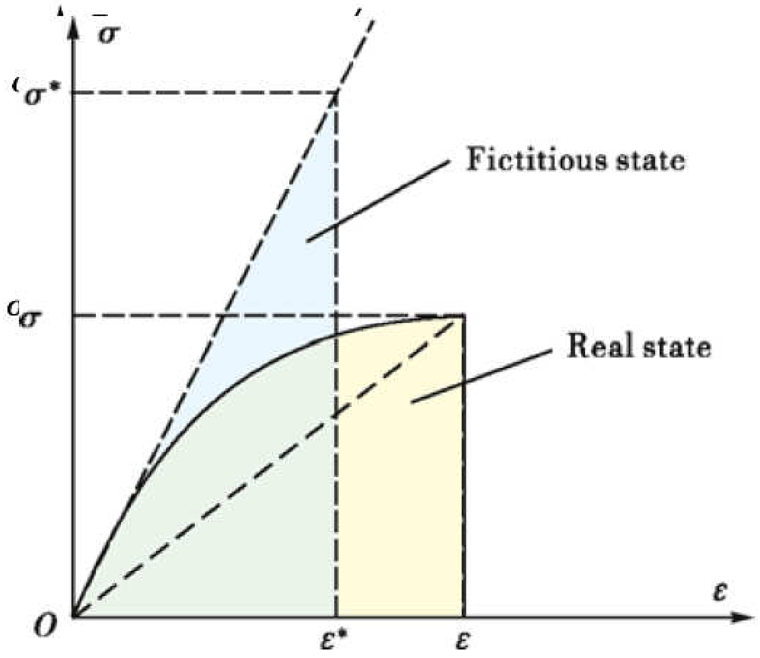
- *The stress associated with a damaged state under the real strain is equivalent to the stress associated with its fictitious undamaged state under the effective strain.*



Hypothesis of Elastic Energy Equivalence

Strain Energy Equivalency

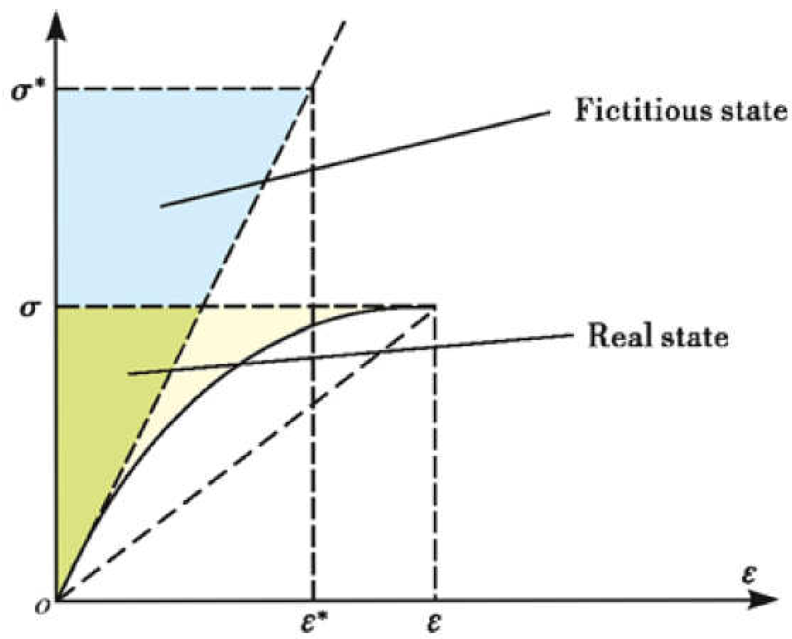
- *The elastic strain energy of a damaged state is assumed to be equal to that of the fictitious undamaged state with the effective strain deformation.*



Hypothesis of Elastic Energy Equivalence

Complementary Energy Equivalency

- *The complementary energy of a damaged state is assumed to be equal to that of the fictitious undamaged state under the effective stress loading.*



Stiffness definition based on strain energy equivalence principles

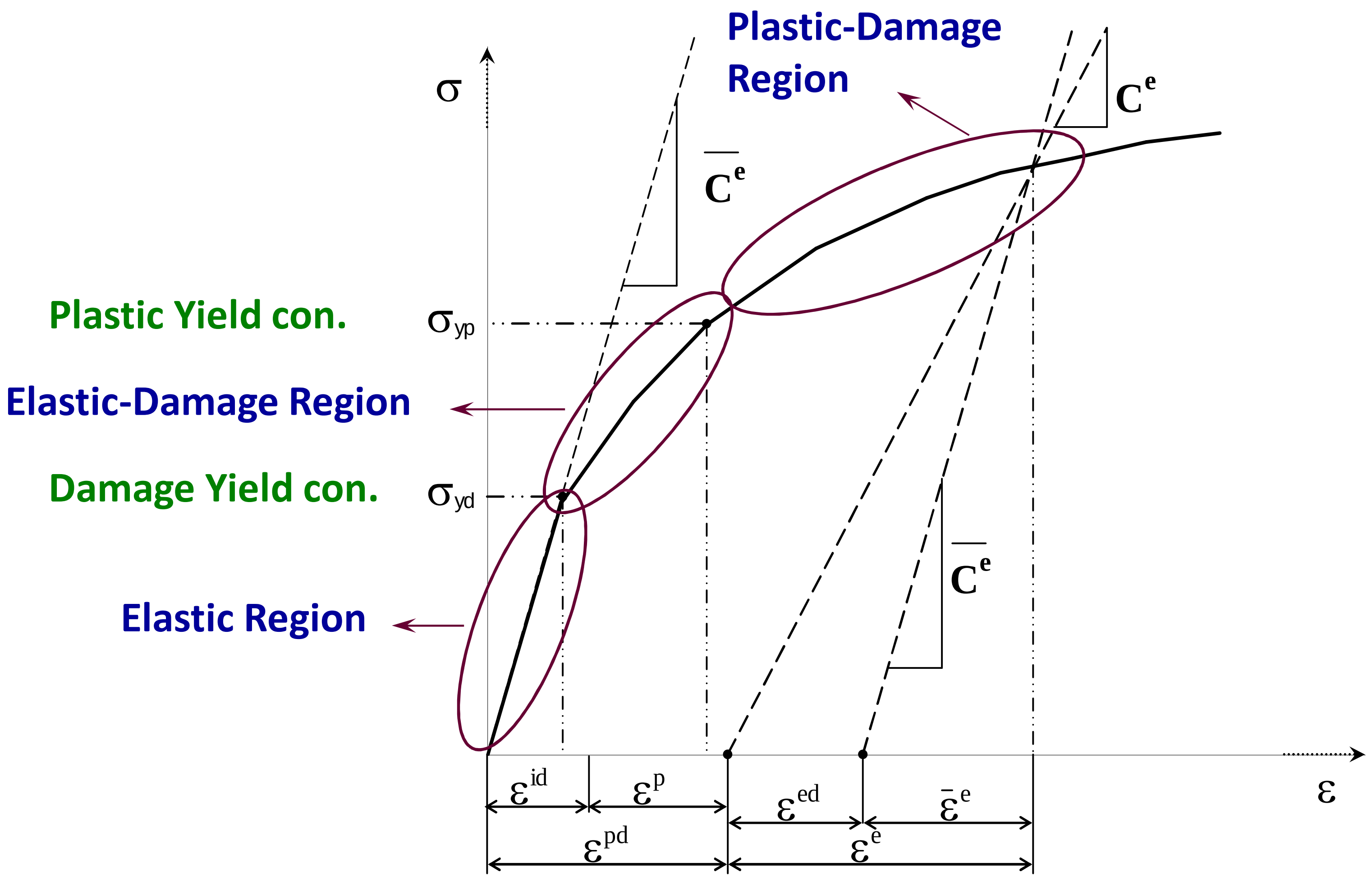
$$\left\{\begin{array}{l}\overline{\sigma} = M(\varphi) : \sigma \\ \\ \frac{1}{2}\overline{\sigma}_{ij}\overline{\varepsilon}_{ij} = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}\end{array}\right.$$

$$\Downarrow$$

$$\mathbf{C}^e(\varphi) = \mathbf{M}^{-1} : \overline{\mathbf{C}}^e : \mathbf{M}^{-T}$$

Effects of Elastic-Plastic Damage Condition on Stress-Strain Constitutive Law

Typical stress-strain curve for a material with damage and Plasticity is shown in this figure



State laws in the framework of irreversible thermodynamics

$$\sigma = \rho \frac{\partial \psi}{\partial \varepsilon^e} = \mathbf{C}^e(\varphi) : \varepsilon^e$$

$$Y = -\frac{\partial \psi}{\partial \varphi} = -\frac{1}{2} \varepsilon^{eT} \frac{\partial C^e(\varphi)}{\partial \varphi} \varepsilon^e$$

$$K = -\frac{\partial \psi}{\partial \kappa} = -c_1^d \left[1 - \exp\left(\kappa / c_2^d\right)\right] \text{ or}$$

$$K = c_1^d \kappa$$

σ is stress tensor

Y is damage conjugate force tensor,

K is isotropic hardening/softening conjugate relation

Using these equations, damage potential and damage evolution laws can be defined.

Analytical expression of the thermodynamic potentials

One of the main issue in the irreversible thermodynamic is definition of thermodynamic potentials.

Helmholtz free energy

$$\rho\psi = E(\varepsilon^{e}, \varphi) + \prod^{d}(\kappa)$$

Strain energy

$$E(\varepsilon^{e}, \varphi) = \frac{1}{2}\varepsilon^{eT} : \mathbf{C}^{e}(\varphi) : \varepsilon^{e}$$

Dissipation energy

$$\begin{cases} \prod^{d}(\kappa) = c_1^{d}\left[c_2^{d} \exp\left(\kappa / c_2^{d}\right) - \kappa\right]; \quad \text{or} \\ \prod^{d}(\kappa) = \frac{1}{2} c_1^{d} \kappa^{2} \end{cases}$$

State laws in the framework of irreversible thermodynamics

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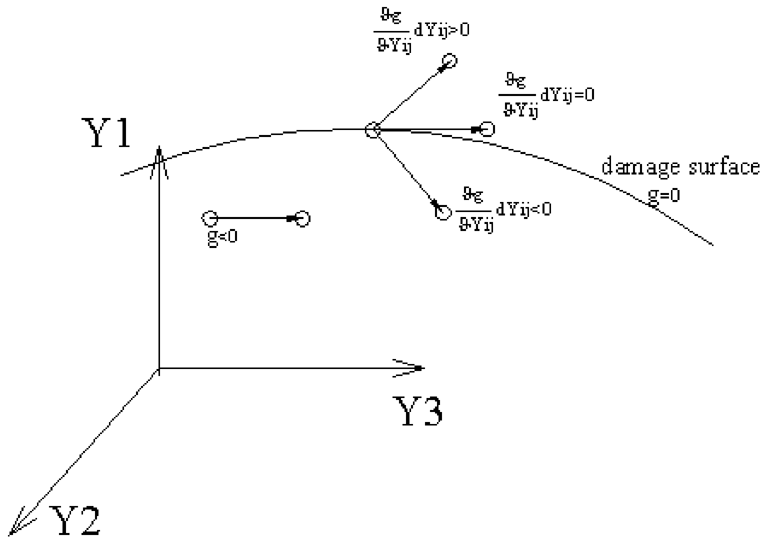
Damage Criteria



Analogous to plasticity, damaging behaviour can be distinguished from non-damaging behaviour by a criteria as follows:

$$G = g(\mathbf{Y}, \kappa) = \sqrt{\mathbf{Y.J.Y}} - (K(\kappa) - K_0)$$

which:

Tensor of J is a material dependent tensor which is determined here by available experimental data on a single lamina,

and, K_0 is the initial damage threshold at which damage begins to occur

Damage Criterion:

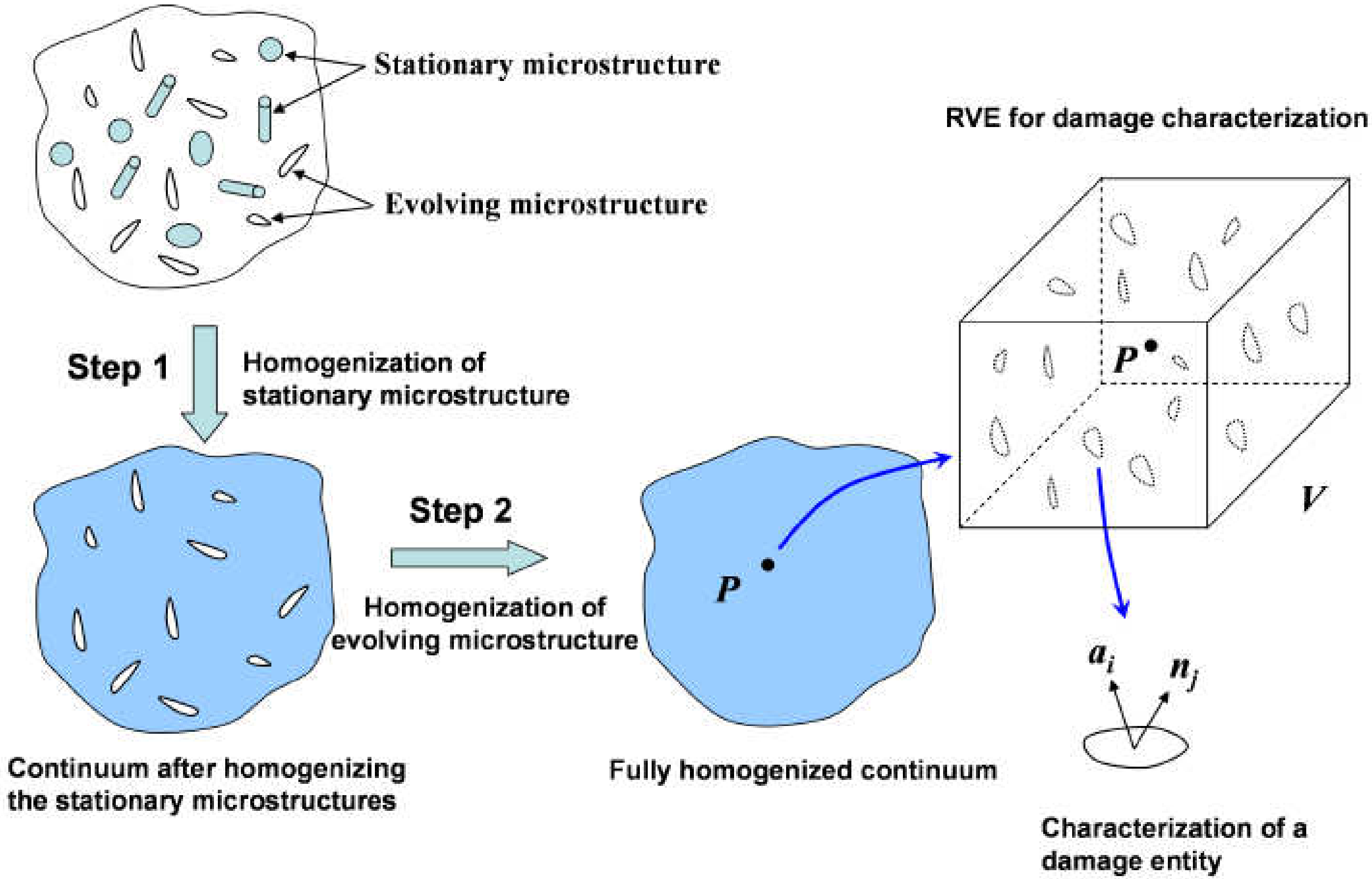
g(Y,k) < 0 indicates a non-damaging state,

g(Y, k)=0 indicates a damage inducing state,

g(Y, k)>0 is understood to be inadmissible

The CDM concept for composites

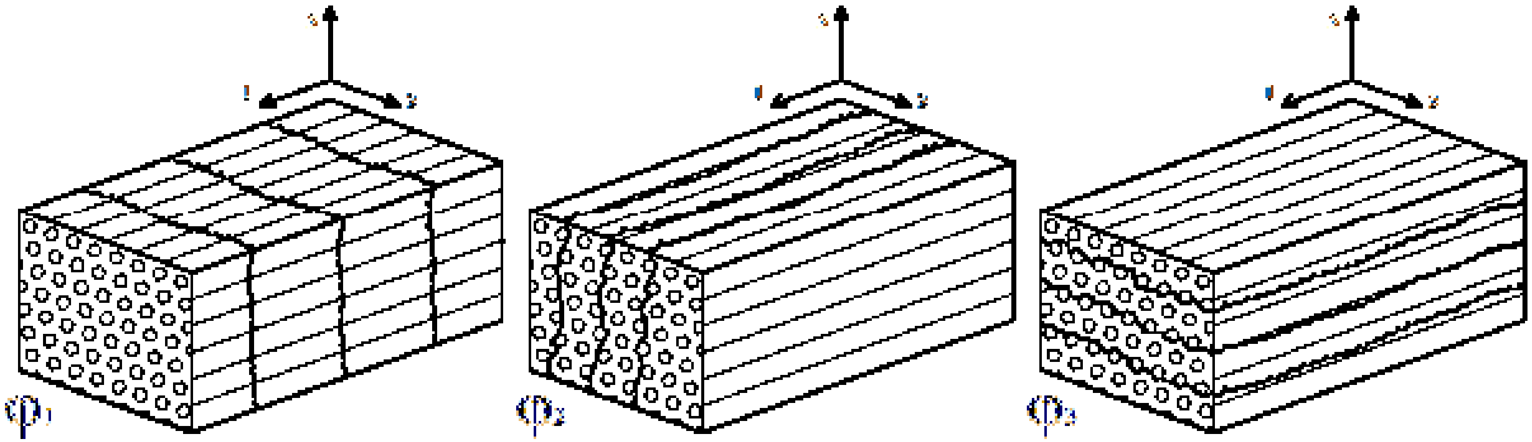
- Illustration of the two-step homogenization process for composites with damage



Implementation of CDM to the homogenized Composites

In the laminated composites and in the ply level:

- **Fiber breakage, fiber/matrix debonding, and matrix cracking are oriented either parallel to the fibers direction or perpendicular to that.**
- **Distribution of above mentioned damages, described by damage eigenvalues φ_1, φ_2 and φ_3 normal to the principal material directions.**



Damage Evolution Equations
Using Continuum Damage Mechanics

- **Clausius-Duhem inequality**

$$\prod = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{id} - \mathbf{Y} : \dot{\boldsymbol{\varphi}} - \rho \, \mathrm{K} . \dot{\kappa} - \mathbf{q} . \frac{\nabla \mathrm{T}}{\mathrm{T}} \geq 0$$

- **Using the theory of functions with several variables, damage Lagrange multiplier $\dot{\lambda}^{d}$ is utilized to construct the objective function of Ω.**

$$\Omega = \prod - G \dot{\lambda}^{d}$$

$$\frac{\partial \Omega}{\partial \boldsymbol{\sigma}} = 0; \qquad \frac{\partial \Omega}{\partial \mathbf{Y}} = 0; \qquad \frac{\partial \Omega}{\partial \mathrm{K}} = 0;$$

Extremize conditions

$$\dot{\boldsymbol{\varepsilon}}^{id} = + \frac{\partial G}{\partial \boldsymbol{\sigma}} \dot{\lambda}^{d}; \qquad \dot{\boldsymbol{\varphi}} = - \frac{\partial G}{\partial \mathbf{Y}} \dot{\lambda}^{d}; \qquad \dot{\kappa} = - \frac{\partial G}{\partial \mathrm{K}} \dot{\lambda}^{d};$$

Evolution equations

$$\dot{\lambda}^{d} \geq 0; \qquad G \leq 0; \qquad \dot{\lambda}^{d} G = 0;$$

Kuhn–Tucker optimality conditions

Stress Integration Algorithm

Return Mapping Algorithm

$$g^{(k)} + \frac{dg^{(k)}}{d\Delta\lambda_j^{d(k)}} d\lambda_j^{d(k)} = 0$$

Linearized form of the damage surface equation

$$\begin{cases} \Delta\boldsymbol{\varepsilon}_j^{id} = g_{,\boldsymbol{\sigma}_j} \Delta\lambda_j^d \\ \Delta\varphi_j = -g_{,\mathbf{Y}_j} \Delta\lambda_j^d \\ \Delta\kappa_j = -g_{,\kappa_j} \Delta\lambda_j^d \end{cases}$$

Evolution equations for state variables

These 2 set of nonlinear equations must be satisfied.

$$\begin{cases} \tilde{\mathbf{e}}_j - \boldsymbol{\varepsilon}_j^{id} + \boldsymbol{\varepsilon}_{j-1}^{id} + g_{,\boldsymbol{\sigma}_j} \Delta\lambda_j^d = 0 \\ g_j = g\left(\mathbf{Y}_j, \kappa_j\right) = \sqrt{\mathbf{Y}_j . \mathbf{J} . \mathbf{Y}_j} - \left(K\left(\kappa_j\right) - K_0\right) = 0 \end{cases}$$

Updating the state variables

$$\Delta\lambda_j^{d(k+1)} = \Delta\lambda_j^{d(k)} + d\lambda_j^{d(k)}$$

$$\boldsymbol{\sigma}_j^{(k+1)} = \boldsymbol{\sigma}_j^{(k)} + d\,\boldsymbol{\sigma}_j^{(k)}$$

$$\kappa_j^{(k+1)} = \kappa_j^{(k)} - g_{,\kappa_j}^{(k)} d\lambda_j^{d(k)}$$

$$\boldsymbol{\varphi}_j^{(k+1)} = \boldsymbol{\varphi}_j^{(k)} - g_{,\mathbf{Y}_j}^{(k)} d\lambda_j^{d(k)} - dg_{,\mathbf{Y}_j}^{(k)} \Delta\lambda_j^{d(k)}$$

$$\boldsymbol{\varepsilon}_j^{id(k+1)} = \boldsymbol{\varepsilon}_j^{id(k)} + g_{,\sigma_j}^{(k)} d\lambda_j^{d(k)} + dg_{,\sigma_j}^{(k)} \Delta\lambda_j^{d(k)}$$