Damage Mechanics in Composite Materials

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Chapter 3

Progressive Damage Analysis

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Maximum Stress Criteria

$$e_1^t = \frac{\sigma_{11}}{X_T} \text{ for } \sigma_{11} \geq 0; \quad e_1^c = \frac{|\sigma_{11}|}{X_C} \text{ for } \sigma_{11} \leq 0$$

$$e_2^t = \frac{\sigma_{22}}{Y_T} \text{ for } \sigma_{22} \geq 0; \quad e_2^c = \frac{|\sigma_{22}|}{Y_C} \text{ for } \sigma_{22} \leq 0$$

$$e_3^t = \frac{\sigma_{33}}{Z_T} \text{ for } \sigma_{33} \geq 0; \quad e_3^c = \frac{|\sigma_{33}|}{Z_C} \text{ for } \sigma_{33} \leq 0$$

$$e_4 = \frac{|\tau_{12}|}{S_{12}}; \quad e_5 = \frac{|\tau_{23}|}{S_{23}}; \quad e_6 = \frac{|\tau_{13}|}{S_{13}}$$

Maximum Strain Criteria

$$e_1^t = \frac{\varepsilon_{11}}{\varepsilon_{11T}^{ult}} \text{ for } \varepsilon_{11} \geq 0; \quad e_1^c = \frac{|\varepsilon_{11}|}{\varepsilon_{11C}^{ult}} \text{ for } \varepsilon_{11} \leq 0$$

$$e_2^t = \frac{\varepsilon_{22}}{\varepsilon_{22T}^{ult}} \text{ for } \varepsilon_{22} \geq 0; \quad e_2^c = \frac{|\varepsilon_{22}|}{\varepsilon_{22C}^{ult}} \text{ for } \varepsilon_{22} \leq 0$$

$$e_3^t = \frac{\varepsilon_{33}}{\varepsilon_{33T}^{ult}} \text{ for } \varepsilon_{33} \geq 0; \quad e_3^c = \frac{|\varepsilon_{33}|}{\varepsilon_{33C}^{ult}} \text{ for } \varepsilon_{33} \leq 0$$

$$e_4 = \frac{|\gamma_{12}|}{\gamma_{12}^{ult}}; \quad e_5 = \frac{|\gamma_{23}|}{\gamma_{23}^{ult}}; \quad e_6 = \frac{|\gamma_{13}|}{\gamma_{13}^{ult}}$$

Tsai-Wu Failure Polynomial

$$\phi = F_1\sigma_{11} + F_2\sigma_{22} + F_3\sigma_{33} + F_{11}(\sigma_{11})^2 + F_{22}(\sigma_{22})^2 + F_{33}(\sigma_{33})^2$$
$$+ 2F_{12}\sigma_{11}\sigma_{22} + 2F_{23}\sigma_{22}\sigma_{33} + 2F_{13}\sigma_{11}\sigma_{33}$$
$$+ F_{44}(\sigma_{13})^2 + F_{55}(\sigma_{23})^2 + F_{66}(\sigma_{12})^2$$

$$F_1 = \frac{1}{X_T} - \frac{1}{X_C}; \quad F_2 = \frac{1}{Y_T} - \frac{1}{Y_C}; \quad F_3 = \frac{1}{Z_T} - \frac{1}{Z_C}$$

$$F_{11} = \frac{1}{X_T X_C}; \quad F_{22} = \frac{1}{Y_T Y_C}; \quad F_{33} = \frac{1}{Z_T Z_C}$$

$$\mathrm{F}_{44} = \frac{1}{(S_{13})^2}; \quad \mathrm{F}_{55} = \frac{1}{(S_{23})^2}; \quad \mathrm{F}_{66} = \frac{1}{(S_{12})^2}$$

$$F_{12} = -\frac{1}{2}\frac{1}{\sqrt{X_T X_C Y_T Y_C}}; \quad F_{13} = -\frac{1}{2}\frac{1}{\sqrt{X_T X_C Z_T Z_C}}$$

$$F_{23} = -\frac{1}{2}\frac{1}{\sqrt{Y_T Y_C Z_T Z_C}}$$

- Since the Tsai-Wu failure model is an *interacting failure model that provides only a single condition for local material failure (i.e.,* where ϕ*>1 denotes failure initiation), identifying the mode of failure requires a different* approach for the material degradation step in the progressive failure model from that used with the maximum stress criteria.

$$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 = \sum_{i=1}^{6} \phi_i = \begin{cases} \leq 1 \ldots \text{No failure} \\ > 1 \ldots \text{Failure} \end{cases}$$

$$\phi_1 = F_1\sigma_{11} + F_{11}(\sigma_{11})^2 + F_{12}\sigma_{11}\sigma_{22} + F_{13}\sigma_{11}\sigma_{33}$$

$$\phi_2 = F_2\sigma_{22} + F_{22}(\sigma_{22})^2 + F_{12}\sigma_{11}\sigma_{22} + F_{23}\sigma_{22}\sigma_{33}$$

$$\phi_3 = F_3\sigma_{33} + F_{33}(\sigma_{33})^2 + F_{13}\sigma_{11}\sigma_{33} + F_{23}\sigma_{22}\sigma_{33}$$

$$\phi_4 = F_{66}(\sigma_{12})^2$$

$$\phi_5 = F_{44}(\sigma_{13})^2$$

$$\phi_6 = F_{55}(\sigma_{23})^2$$

$$e_i = \phi_i \quad \text{for } i = 1, 2, \ldots, 6 \quad \text{when} \quad \phi \geq 1$$

- The largest value of the failure indices *ei defines the failure mode as* i_{max}.
- No distinction is made between tension and compression values for the normal stress components in evaluating the Tsai-Wu failure polynomial terms.

Hashin Failure Criteria

- Tensile fiber failure – for $\sigma_{11} \geq 0$

$$\left(e_1^t\right)^2 = \left(\frac{\sigma_{11}}{X_T}\right)^2 + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} = \begin{cases} >1 & \text{failure} \\ \leq 1 & \text{no failure} \end{cases}$$

- Compressive fiber failure – for $\sigma_{11} < 0$

$$\left(e_1^c\right)^2 = \left(\frac{\sigma_{11}}{X_C}\right)^2 = \begin{cases} >1 & \text{failure} \\ \leq 1 & \text{no failure} \end{cases}$$

Hashin Failure Criteria

- Tensile matrix failure – for $\sigma_{22} + \sigma_{33} > 0$

$$\left(e_2^t\right)^2 = \frac{\left(\sigma_{22} + \sigma_{33}\right)^2}{Y_T^2} + \frac{\left(\sigma_{23}^2 - \sigma_{22}\sigma_{33}\right)}{S_{23}^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} = \begin{cases} >1 & \text{failure} \\ \leq 1 & \text{no failure} \end{cases}$$

- Compressive matrix failure – for $\sigma_{22} + \sigma_{33} < 0$

$$\left(e_2^c\right)^2 = \left[\left(\frac{Y_C}{2S_{23}}\right)^2 - 1\right]\left(\frac{\sigma_{22} + \sigma_{33}}{Y_C}\right) + \frac{\left(\sigma_{22} + \sigma_{33}\right)^2}{4S_{23}^2}$$

$$+ \frac{\left(\sigma_{23}^2 - \sigma_{22}\sigma_{33}\right)}{S_{23}^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{S_{12}^2} = \begin{cases} >1 & \text{failure} \\ \leq 1 & \text{no failure} \end{cases}$$

Hashin Failure Criteria

- Interlaminar normal tensile failure – for $\sigma_{33} > 0$

$$\left(e_3^t\right)^2 = \left(\frac{\sigma_{33}}{Z_T}\right)^2 = \begin{cases} >1 & \text{failure} \\ \leq 1 & \text{no failure} \end{cases}$$

- Interlaminar normal compression failure – for $\sigma_{33} < 0$

$$\left(e_3^c\right)^2 = \left(\frac{\sigma_{33}}{Z_C}\right)^2 = \begin{cases} >1 & \text{failure} \\ \leq 1 & \text{no failure} \end{cases}$$

Hashin Failure Criteria

- If any failure index $ei^{t,c}$ *exceeds unity, then failure initiation has occurred for that strain* component at that material point and material degradation will be performed. That is, each failure mode may fail independently at different load levels

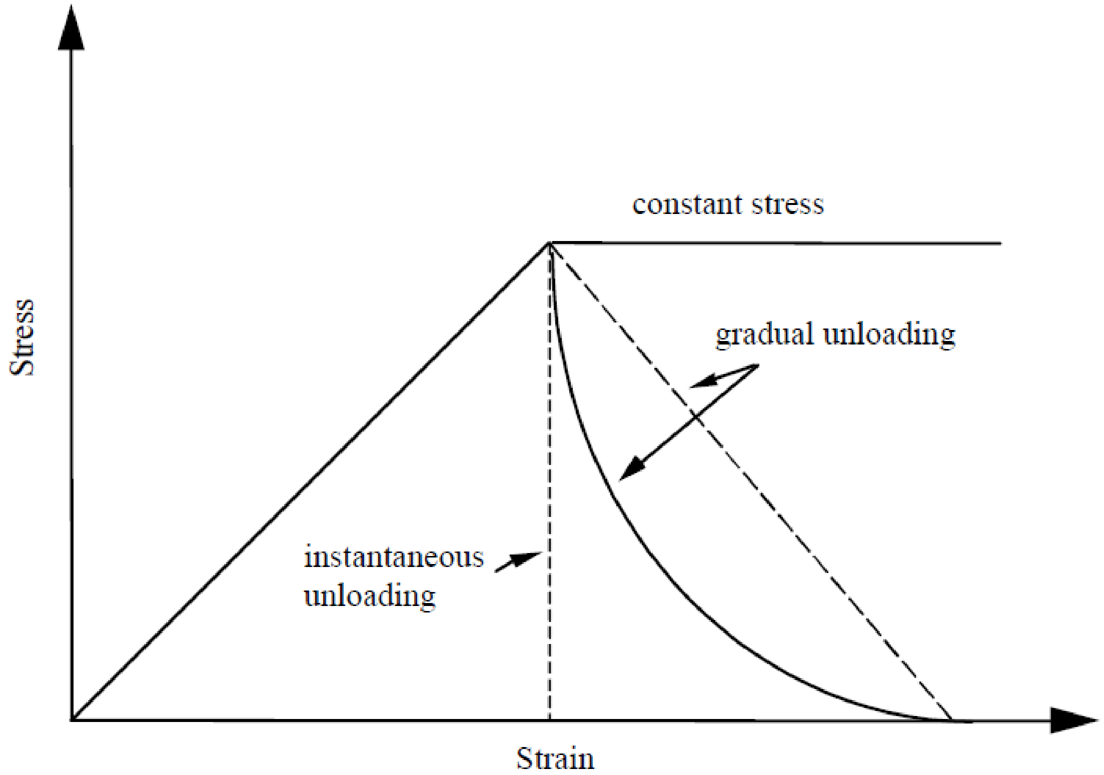
Quadratic Polynomial Failure Criteria

	Quadratic Polynomial Failure Criteria				
	Tsai-Wu	Tsai-Hill*	Azzi-Tsai*	Hoffman	Chamis*†
F_1	$\frac{1}{X_T}-\frac{1}{X_C}$	0	0	$\frac{1}{X_T}-\frac{1}{X_C}$	0
F_2	$\frac{1}{Y_T}-\frac{1}{Y_C}$	0	0	$\frac{1}{Y_T}-\frac{1}{Y_C}$	0
F_3	$\frac{1}{Z_T}-\frac{1}{Z_C}$	0	0	$\frac{1}{Z_T}-\frac{1}{Z_C}$	0
F_{12}	$-1/2\sqrt{X_T X_C Y_T Y_C}$	$-\frac{1}{2}\left(\frac{1}{X^2}+\frac{1}{Y^2}-\frac{1}{Z^2}\right)$	$-\frac{1}{X^2}$	$-\frac{1}{2}\left(\frac{1}{X_T X_C}+\frac{1}{Y_T Y_C}-\frac{1}{Z_T Z_C}\right)$	$-\frac{K_{12}}{XY}$
F_{13}	$-1/2\sqrt{X_T X_C Z_T Z_C}$	$-\frac{1}{2}\left(\frac{1}{Z^2}+\frac{1}{X^2}-\frac{1}{Y^2}\right)$	0	$-\frac{1}{2}\left(\frac{1}{X_T X_C}+\frac{1}{Z_T Z_C}-\frac{1}{Y_T Y_C}\right)$	$-\frac{K_{13}}{XZ}$
F_{23}	$-1/2\sqrt{Y_T Y_C Z_T Z_C}$	$-\frac{1}{2}\left(\frac{1}{Y^2}+\frac{1}{Z^2}-\frac{1}{X^2}\right)$	0	$-\frac{1}{2}\left(\frac{1}{Z_T Z_C}+\frac{1}{Y_T Y_C}-\frac{1}{X_T X_C}\right)$	$-\frac{K_{23}}{YZ}$
F_{11}	$\frac{1}{X_T X_C}$	$\frac{1}{X^2}$	$\frac{1}{X^2}$	$\frac{1}{X_T X_C}$	$\frac{1}{X^2}$
F_{22}	$\frac{1}{Y_T Y_C}$	$\frac{1}{Y^2}$	$\frac{1}{Y^2}$	$\frac{1}{Y_T Y_C}$	$\frac{1}{Y^2}$
F_{33}	$\frac{1}{Z_T Z_C}$	$\frac{1}{Z^2}$	0	$\frac{1}{Z_T Z_C}$	$\frac{1}{Z^2}$
F_{44}	$\frac{1}{R^2}$	$\frac{1}{R^2}$	0	$\frac{1}{R^2}$	$\frac{1}{R^2}$
F_{55}	$\frac{1}{S^2}$	$\frac{1}{S^2}$	0	$\frac{1}{S^2}$	$\frac{1}{S^2}$
F_{66}	$\frac{1}{T^2}$	$\frac{1}{T^2}$	$\frac{1}{T^2}$	$\frac{1}{T^2}$	$\frac{1}{T^2}$

* X, Y, and Z are either X_C, Y_C, and Z_C or X_T, Y_T, and Z_T depending upon the sign of S_1, s_2 and s_3 respectively.

† *K_{12}, K_{13}, and K_{23} are the strength coefficients depending upon material.*

Post-failure degradation behavior in composite laminates



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Options for Material Degradation for Maximum Strain Criterion

Table 2. Options for Material Degradation for Maximum Strain Criterion

	Option 1		Option 2		Option 3	
Primary Failure Direction	Induced Additional Failure	Degraded Properties	Induced Additional Failure	Degraded Properties	Induced Additional Failure	Degraded Properties
Fiber Failure	None	E_{11}, υ_{12}	Shear	$E_{11}, G_{12}, \upsilon_{12}$	Shear	$E_{11}, G_{12}, \upsilon_{12}$
Matrix Failure	None	E_{22}, υ_{12}	Shear	$E_{22}, G_{12}, \upsilon_{12}$	Shear	$E_{22}, G_{12}, \upsilon_{12}$
Shear Failure	None	G_{12}	Matrix	$E_{22}, G_{12}, \upsilon_{12}$	Fiber	$E_{11}, G_{12}, \upsilon_{12}$

Material Degradation for Hashin's and Christensen's Failure Criteria

Table 3. Material Degradation for Hashin's and Christensen's Failure Criteria

	Hashin's Criterion		Christensen's Criterion	
Primary Failure Direction	Induced Additional Failure	Degraded Properties	Induced Additional Failure	Degraded Properties
Fiber Failure	Shear	$E_{11}, G_{12}, \upsilon_{12}$	Shear	$E_{11}, G_{12}, \upsilon_{12}$
Matrix Failure	Shear	$E_{22}, G_{12}, \upsilon_{12}$	Shear	$E_{22}, G_{12}, \upsilon_{12}$

Material degradation factors for 3D models and various failure modes based on the Max. Stress and max. Strain criteria

Failure index i	Failure mode	Degradation factors for diagonal terms of constitutive matrix (corresponding row and column off-diagonal entries are also degraded and symmetry of the constitutive matrix is enforced)					
		C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}
1 (T)	Longitudinal in-plane tension	$\beta_T(C_{11})^o$					
1 (C)	Longitudinal in-plane compression	$\beta_C(C_{11})^o$					
2 (T)	Transverse in-plane tension		$\beta_T(C_{22})^o$				
2 (C)	Transverse in-plane compression		$\beta_C(C_{22})^o$				
3 (T)	Transverse out-of-plane tension			$\beta_T(C_{33})^o$			
3 (C)	Transverse out-of-plane compression			$\beta_C(C_{33})^o$			
4	In-plane shear (12-plane)				$\beta_S(C_{44})^o$		
5	Transverse shear (13-plane)					$\beta_S(C_{55})^o$	
6	Transverse shear (23-plane)						$\beta_S(C_{66})^o$

Material degradation factors for three-dimensional models and various failure modes based on the Tsai-Wu criteria

Failure index i	Failure mode	Degradation factors for diagonal terms of constitutive matrix (corresponding row and column off-diagonal entries are also degraded and symmetry of the constitutive matrix is enforced)					
		C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}
1 (T)	Longitudinal in-plane tension	$\beta_T(C_{11})^o$					
1 (C)	Longitudinal in-plane compression	$\beta_C(C_{11})^o$					
2 (T)	Transverse in-plane tension		$\beta_T(C_{22})^o$				
2 (C)	Transverse in-plane compression		$\beta_C(C_{22})^o$				
3 (T)	Transverse out-of-plane tension			$\beta_T(C_{33})^o$			
3 (C)	Transverse out-of-plane compression			$\beta_C(C_{33})^o$			
4	In-plane shear (12-plane)				$\beta_S(C_{44})^o$		
5	Transverse shear (13-plane)					$\beta_S(C_{55})^o$	
6	Transverse shear (23-plane)						$\beta_S(C_{66})^o$

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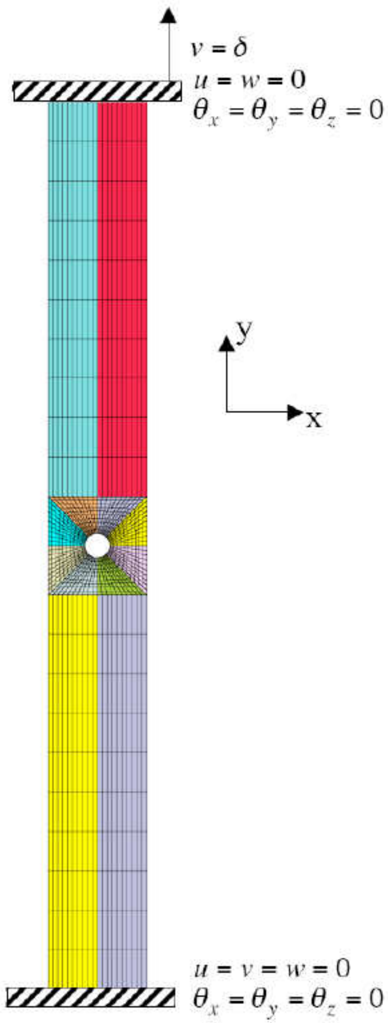
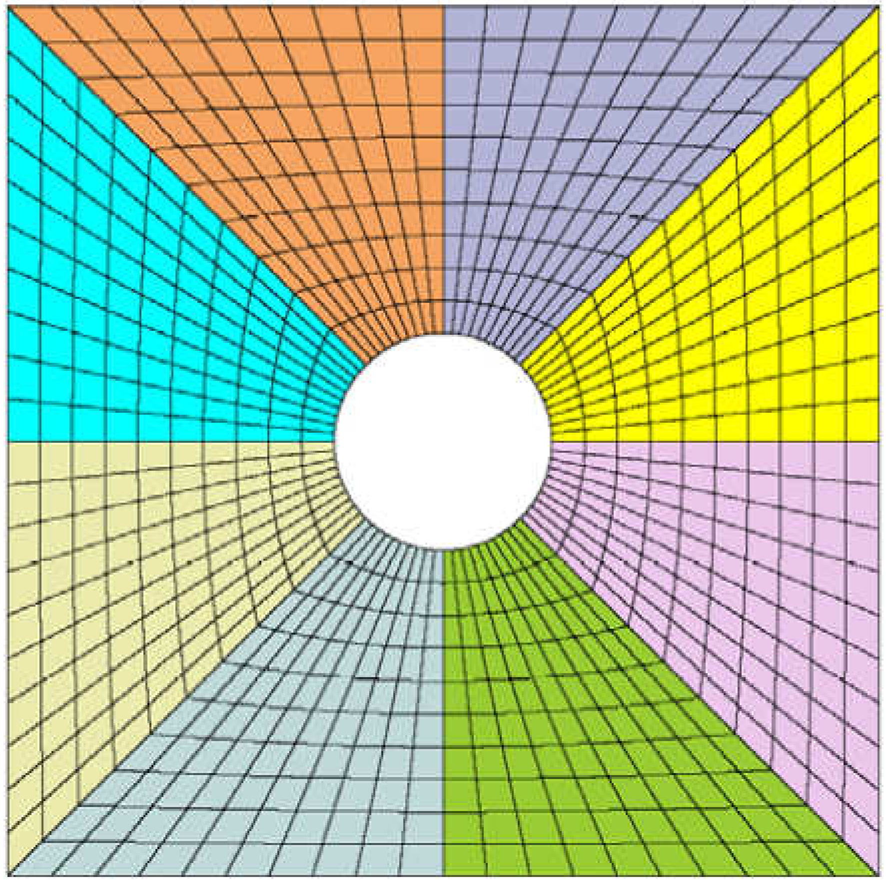
Material degradation factors for 3D models and various failure modes based on the Hashin criteria

Failure index i	Failure mode	Degradation factors for diagonal terms of constitutive matrix (corresponding row and column off-diagonal entries are also degraded and symmetry of the constitutive matrix is enforced)					
		C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}
1 (T)	Longitudinal in-plane tension	$\beta_T(C_{11})^o$			$\beta_S(C_{44})^o$		
1 (C)	Longitudinal in-plane compression	$\beta_C(C_{11})^o$					
2 (T)	Transverse in-plane tension		$\beta_T(C_{22})^o$		$\beta_S(C_{44})^o$		
2 (C)	Transverse in-plane compression		$\beta_C(C_{22})^o$		$\beta_S(C_{44})^o$		
3 (T)	Transverse out-of-plane tension			$\beta_T(C_{33})^o$			
3 (C)	Transverse out-of-plane compression			$\beta_C(C_{33})^o$			
4	In-plane shear (12-plane)				$\beta_S(C_{44})^o$		
5	Transverse shear (13-plane)					$\beta_S(C_{55})^o$	
6	Transverse shear (23-plane)						$\beta_S(C_{66})^o$





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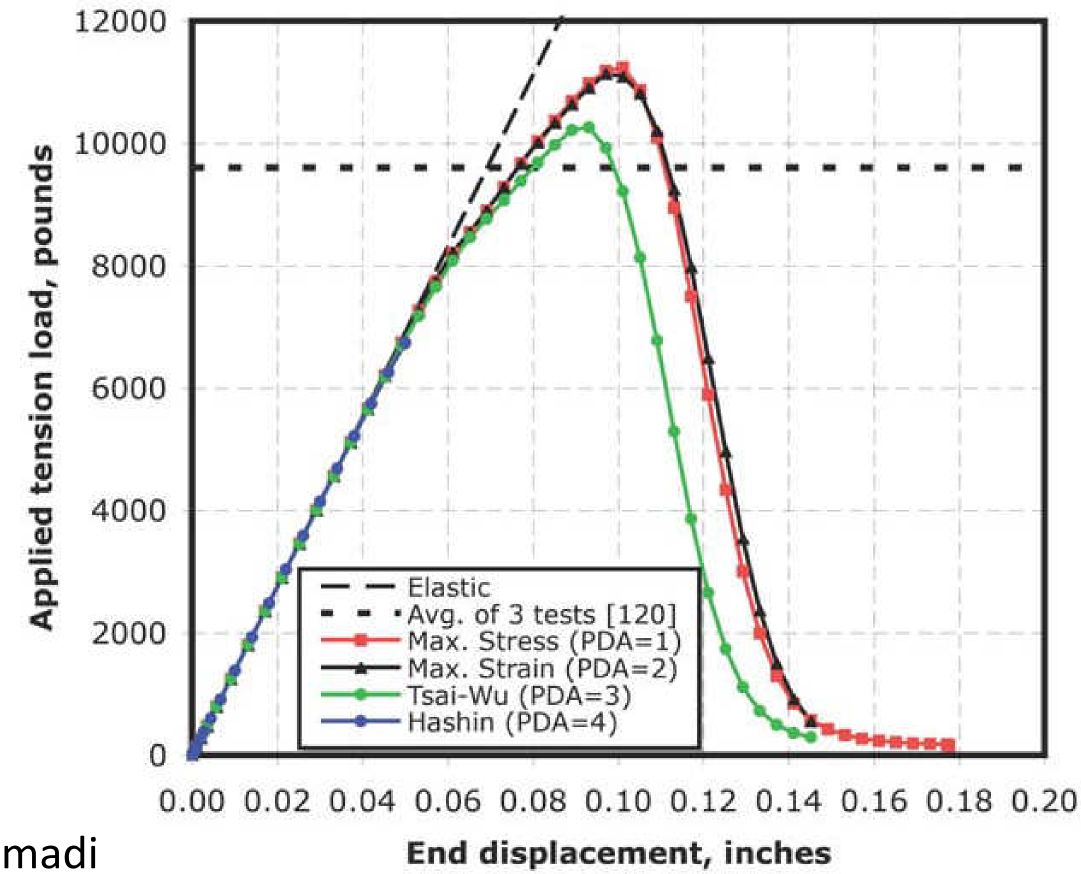
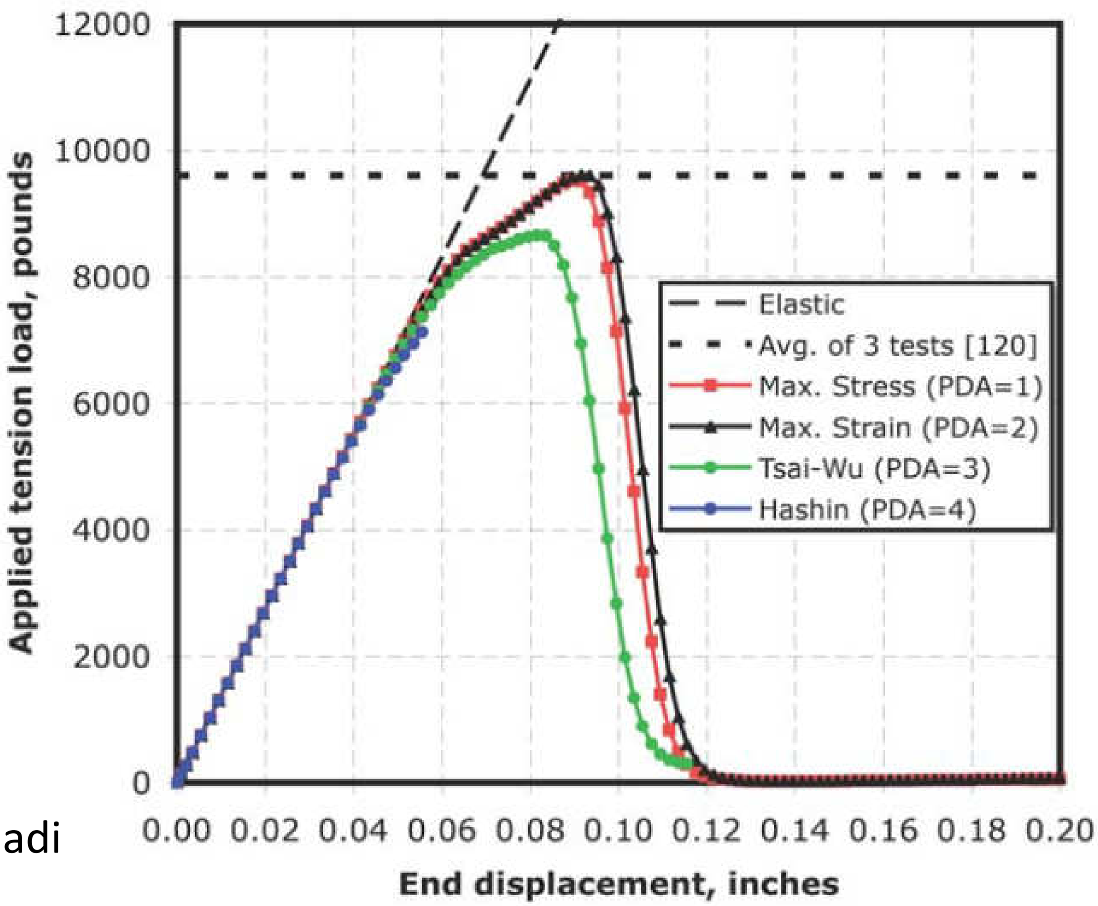
Comparison of different progressive failure analysis models for the 16-ply cross-ply open-hole-tension coupon using recursive degradation with a degradation factor β of 0.5.

- Predictions obtained using a 0.02 value for the maximum solution increment size



Comparison of different progressive failure analysis models for the 16-ply cross-ply open-hole-tension coupon using recursive degradation with a degradation factor β of 0.5.

- Predictions obtained using a 0.01 value for the maximum solution increment size



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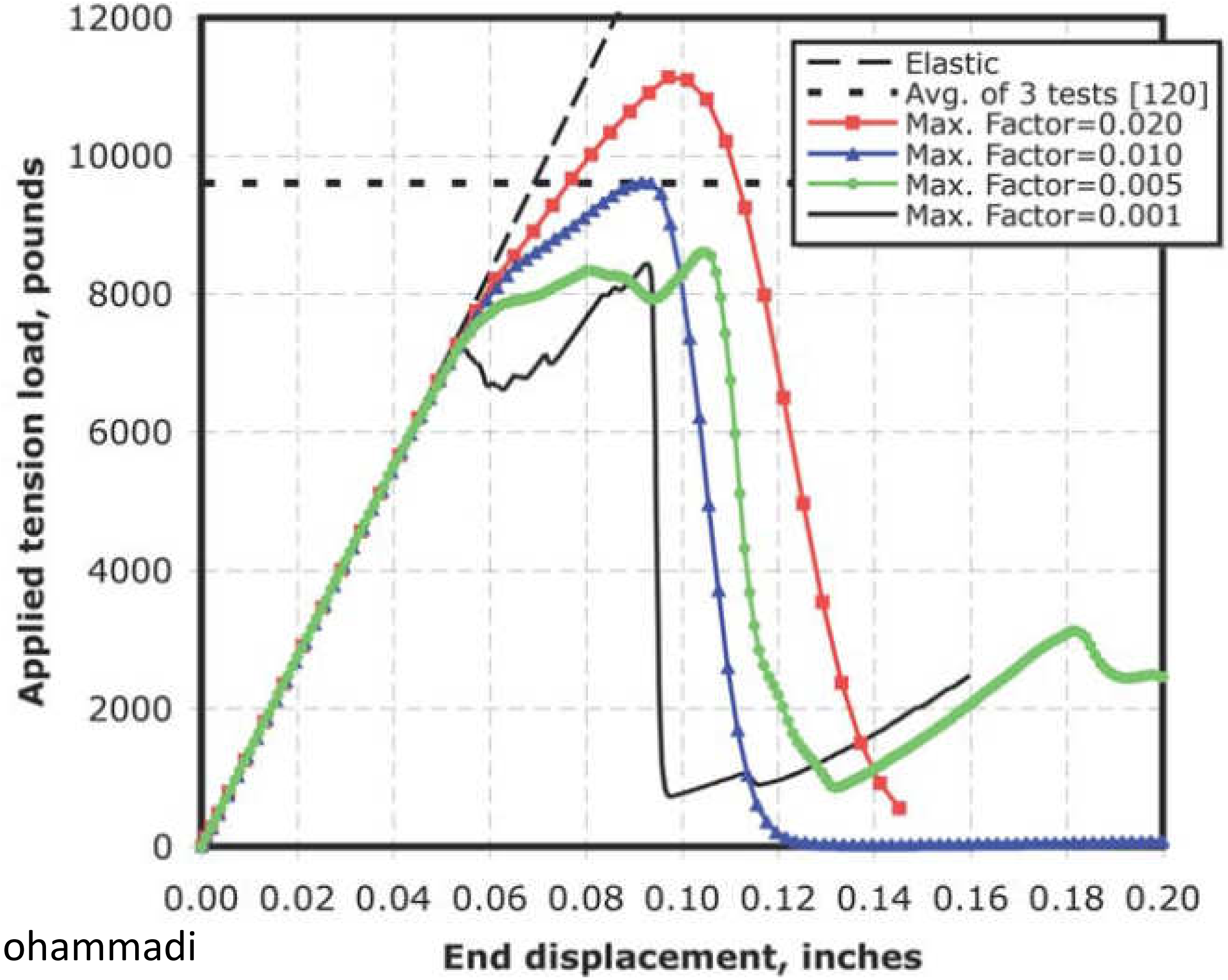
Results using maximum strain criteria



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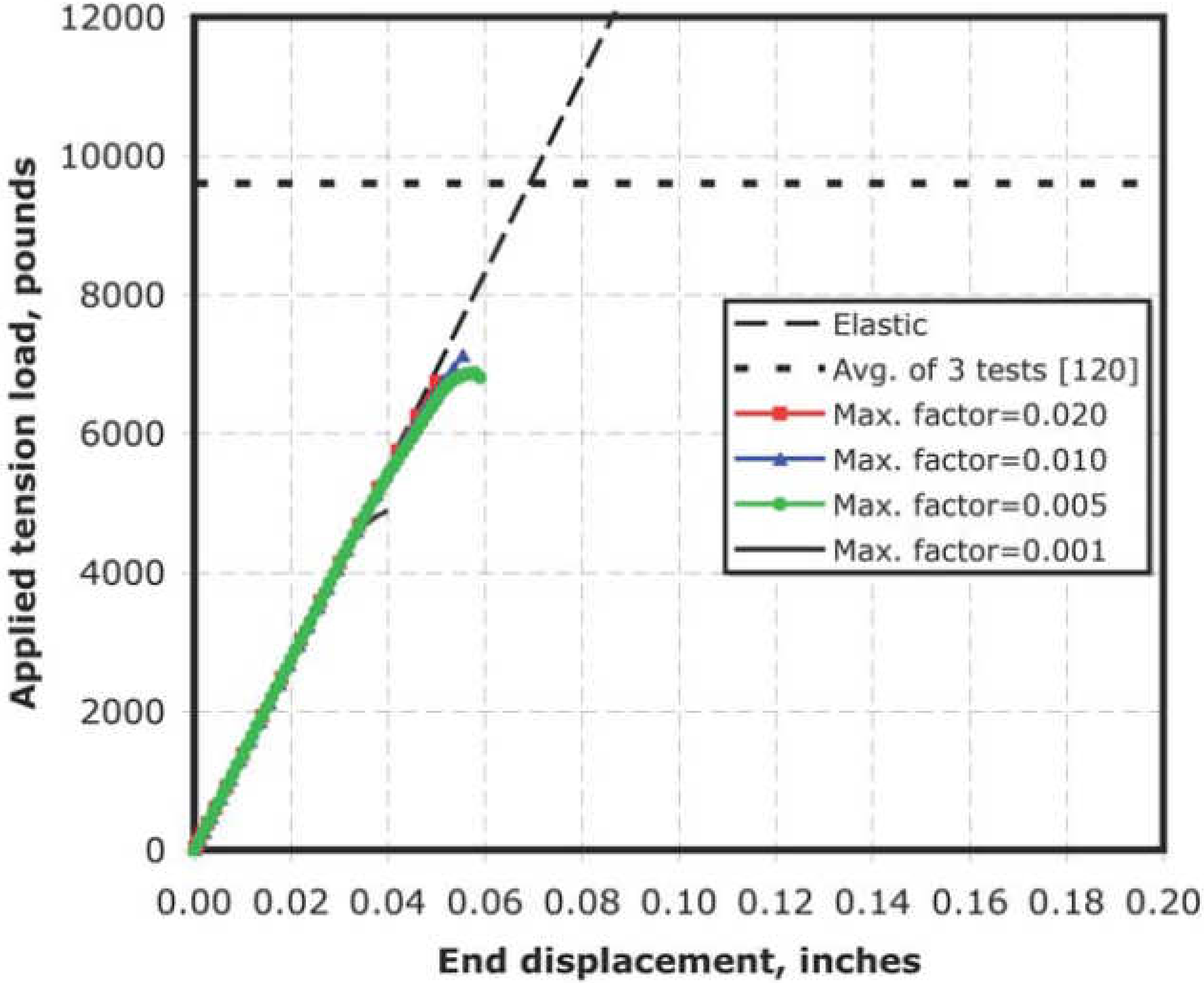
Results using Hashin criteria



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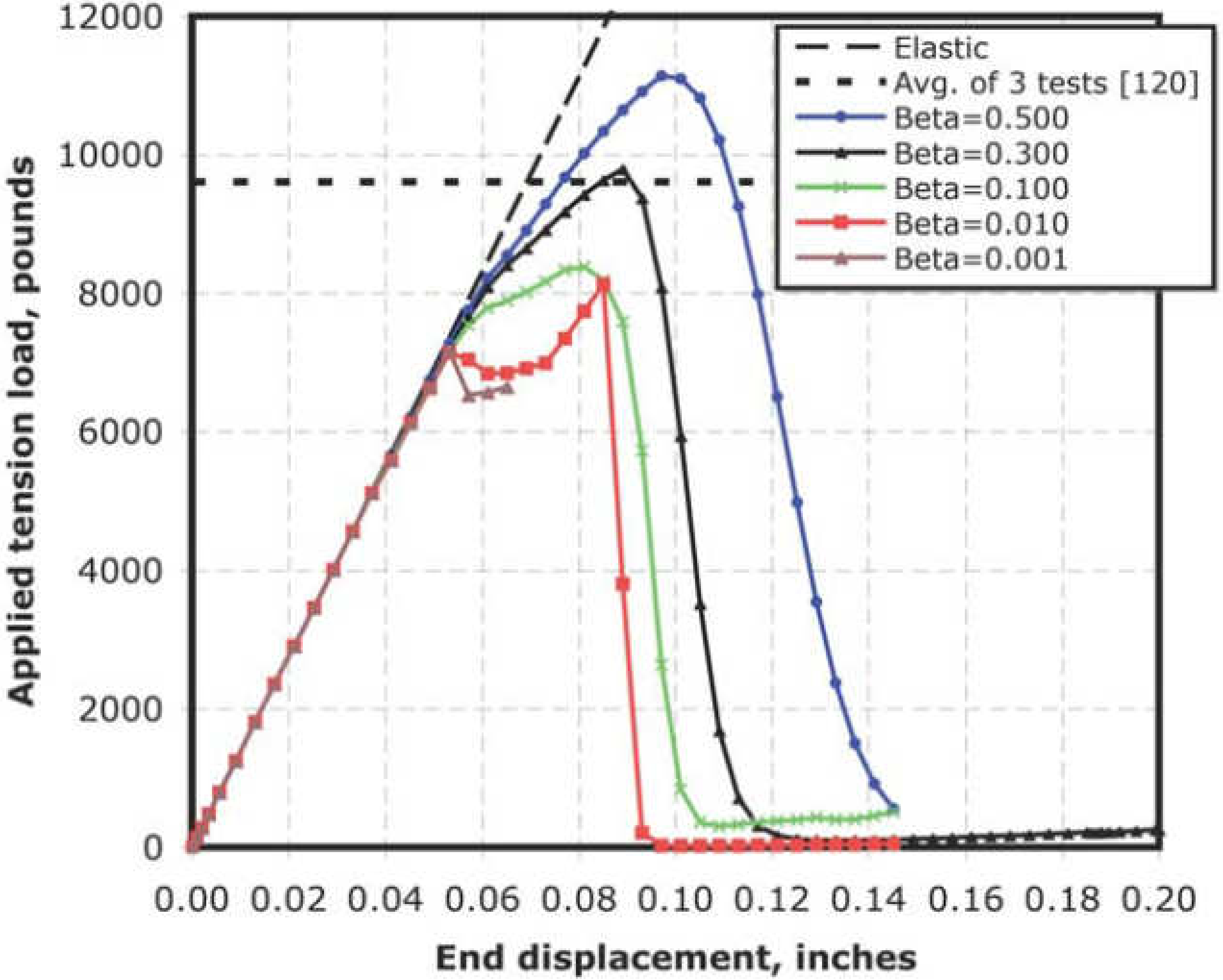
Results using maximum strain criteria



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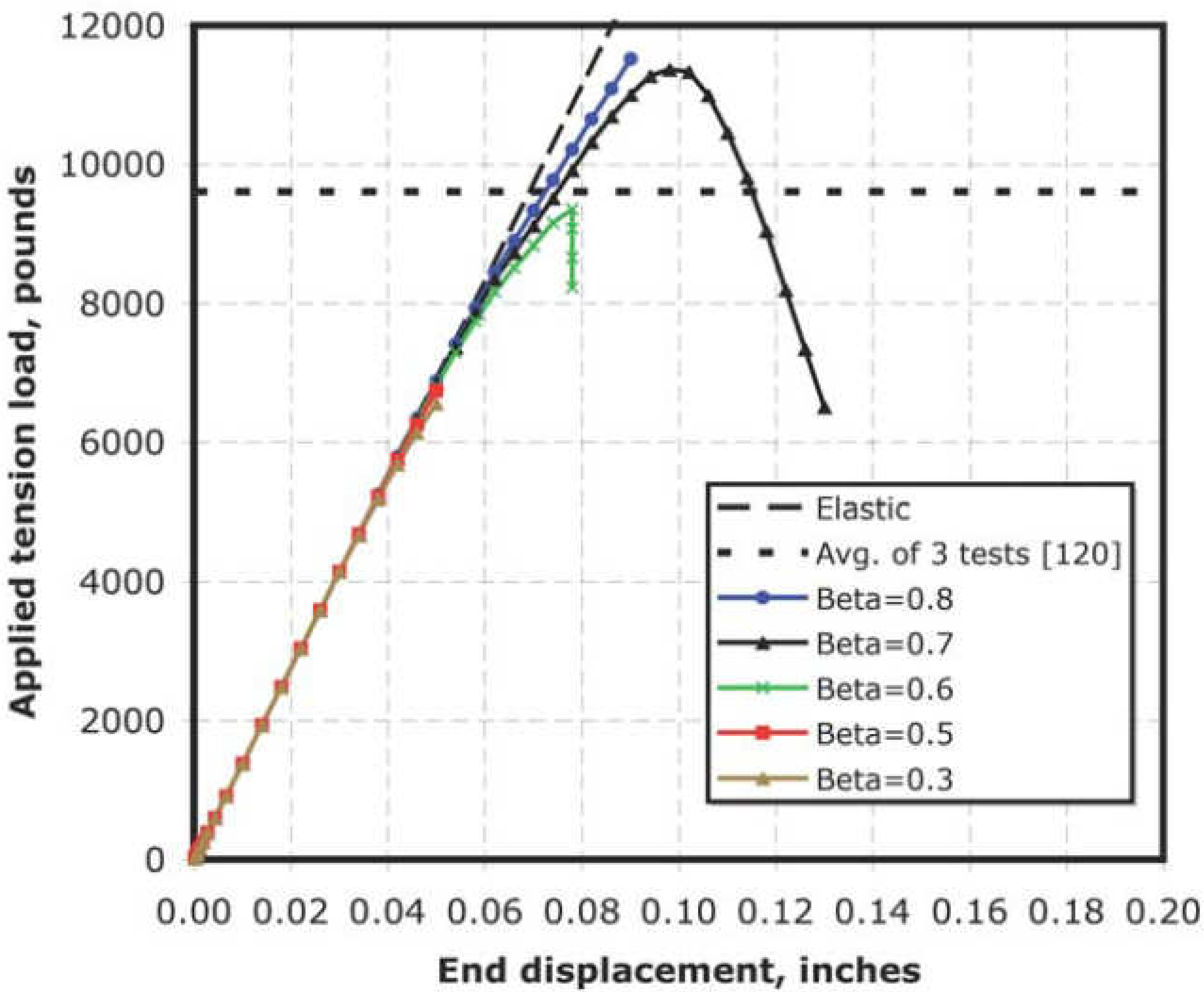
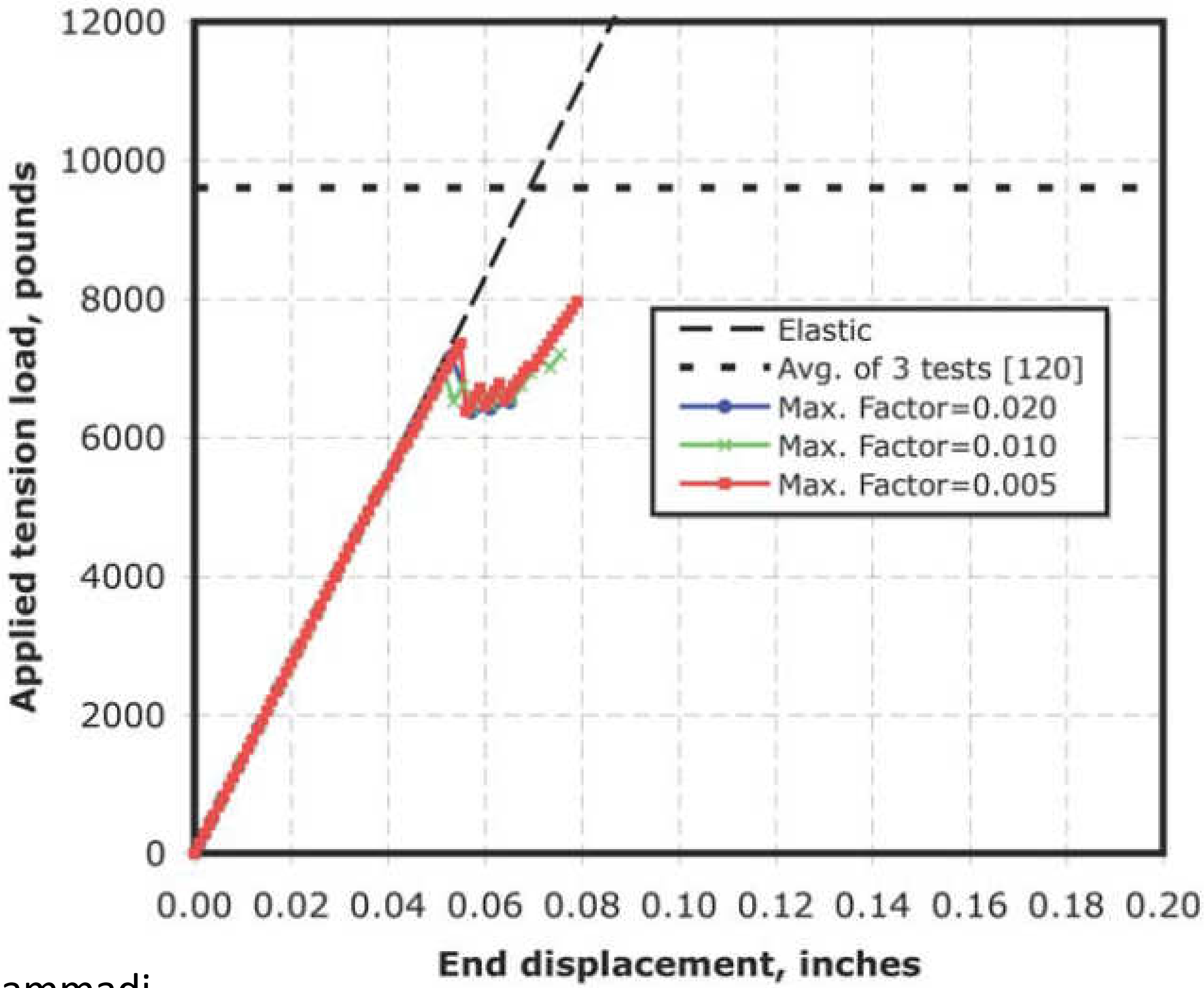
Results using Hashin criteria



Influence of maximum solution increment size on the progressive failure predictions for the 16-ply cross-ply open-hole-tension coupon using the maximum strain criteria and instantaneous degradation with a degradation factor β of *10-6.*



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Comparison of different progressive failure analysis models for the 16-ply quasi-isotropic open-hole-tension coupon using recursive degradation with a degradation factor β of 0.5 and a maximum solution increment size of 0.01.