An investigation on crack growth rate of fatigue and induction heating thermo-mechanical fatigue (TMF) in Hastelloy X superalloy via LEFM, EPFM and integration models

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\textbf{A B S T R A C T}

In this research, fatigue and induction heating thermo-mechanical fatigue (TMF) were performed on Hastelloy X superalloy in the small and large scale yielding in plane stresses mode. The crack growth rates were measured and formulated by fracture mechanics parameters. Furthermore, the fatigue life was predicted by employing resistance curves technique. The TMF behavior of this superalloy was investigated. The results demonstrated that in-phase loading TMF conditions lead to short fatigue life (more crack growth rate) at high strain amplitudes and temperatures up to 600°C. For higher temperatures, the predominant damage was due to creep. A model based on damage contributions due to pure fatigue and cyclic creep has been presented for predicting TMF crack growth rates. Fracture mechanic method was used to suggest a model for fatigue part of TMF crack growth rate, while the temperature effects during TMF crack growth rate was considered to be due to cyclic creep. In addition the TMF crack healing or crack closure occur during application of induced eddy currents were investigated explicitly, as environmental effects of induction heating. The results show the higher current density in the crack tip area produced more heat and resulted in a significant rise in temperature. So it was concluded that the compressive thermal stress due to change of thermal expansion causes crack healing.

\section{1. Introduction}

The gas turbine engine components at high temperatures are exposed to thermo mechanical fatigue (TMF). The Hastelloy X is one of the nickel based super alloys in the hot section parts of engine that possesses an exceptional combination of high-temperature strength, oxidation resistance, fabricability and it has also been found to have exceptionally resistant to stress-corrosion cracking in petrochemical applications [1]. Thermo-mechanical fatigue crack growth is more complicated than pure fatigue crack growth because it includes time dependent material behavior characteristics such as creep and oxidation. According to Antolovich and Saxena [2], TMF, can be viewed as a fatigue damage process under simultaneous changes in temperature and mechanical loads. Some studies have been done on the TMF behavior of nickel based super alloys for fatigue life prediction [3–8]. In most of them, isothermal fatigue (IF), in-phase (IP) and out-of-phase (OP) TMF results have been compared and the strain and damage based approaches were investigated by fracture mechanics parameters such as the stress intensity factor K, the J-integral and the crack-tip opening displacement (CTOD).

Despite the fact that a variety of life prediction models have already been proposed for TMF, environmental effects are mostly not explicitly included. Christ [9] illustrated the variety of environmental, oxygen and hydrogen, influences which can take place during thermo mechanical fatigue of metallic engineering materials, and the resulting effects on TMF life.

In the TMF experiments, heating can be accomplished by various techniques including induction, direct resistance, radiant, or forced air heating. Each of these techniques may be considered as environmental effects. In the current study, induction heating has been accomplished and its effect on the TMF behavior has been investigated. It could be considered as AC magnetic field or eddy current can affect on the TMF crack growth rate. It is known that high density electric current in stainless steel has also been demonstrated [11]. Also one
may consider eddy current treatment as a novel and effective method for crack healing. Healing of Fatigue Crack in 1045 Steel has also been investigated by using Eddy Current Treatment [12]. Crack closure strongly affects the fatigue crack growth rate and thus, the fatigue life of components. Fatigue crack closure under in-phase and out-of-phase thermo mechanical fatigue loading using a temperature dependent strip yield model has been investigated in [13].

In this paper, crack growth rates due to fatigue and induction heating TMF in plates of Hastelloy X superalloy were investigated by fracture mechanic technique. For accomplishing the objectives, room temperature fatigue tests at various stresses were initially applied and then crack growth rates were formulated using stress intensity factor and J-integral parameters. Besides, by extracting the resistance curves, a simple method for prediction of fatigue life has been proposed. The TMF tests in IF, IP and OP were performed in order to investigate TMF behavior of Hastelloy X. In addition, new models for predicting crack growth rate and TMF life on the basis of the amounts of contributions of fatigue and creep on TMF were proposed. Effects of temperature on TMF crack growth rate were considered utilizing fracture mechanic equations. Also fatigue and cyclic creep contributions at various temperatures were presented in a robust mathematical model. Finally, the TMF crack healing and crack closure by induced eddy currents, as environmental effects of induction heating, have been investigated explicitly in this research.

2. Experimental methods

2.1. Material, specimen preparation and apparatuses

The starting material used in this investigation was Hastelloy X, a nickel based superalloy, with a chemical composition, identical to UNS N06602 standard shown in Table 1. It was delivered in the form of plate shape with 2 mm thickness. These plates were heat treated at 1175 °C for 1 h and then rapidly cooled in air.

The compact tension CT specimens, that are single edge-notch specimens loaded in tension, were fabricated with the dimensions given in Fig. 1. It should be mentioned that standard CT specimen, fatigue precracking and other apparatuses such as clevis and pins met the requirements of ASTM E 647 [14]. The specimens with 2 mm thickness then subjected to tensile test in plane stress mode.

In order to measure the crack size and crack opening displacement, COD, an EOS 700D digital camera with 5184 × 3156 pixel resolution and a crack gauge were used. For application of induction heating in thermo mechanical fatigue, TMF, an elliptic coil composed of 7 turns of copper wire was used. The elliptic shape of coil lets to symmetrical heating in two sides of the plate. A power within the range of 1.6–2 kW was used. Water circulation for cooling of copper wires and also grips and fixtures for holding the specimen were used. Fig. 2 shows set up of TMF equipments.

2.2. Experimental procedures

According to fracture mechanics, fatigue crack growth could be investigated by two different scales: linear elastic fracture mechanics (LEFM) and elastic plastic fracture mechanics (EPFM). The ASTM E 647 [14] standard required that the specimen be predominantly elastic at all values of applied force, thus for the CT specimen the following relation is required for LEFM:

\[
\frac{W - a}{l} \geq \left(\frac{4}{\pi}\right)\left(\frac{K_{\text{max}}}{\sigma_{YS}}\right)^2
\]

(1)

where: \(W\) is the total ligament length, \(a\) is the crack length and \(W - a = \text{the length of specimen's uncracked ligament (see Fig. 1)}\), and \(\sigma_{YS} = 0.2\%\) offset yield strength determined at the same temperature that fatigue crack growth rate data were taken. Here \(K_{\text{max}}\) is the maximum stress intensity factor. For the CT specimen one can calculate \(\Delta K\) by using the following relationship [14]:

\[
\Delta K = \frac{\Delta P}{B\sqrt{W}} \left(\frac{2 + \alpha}{1 - \alpha}\right) \left(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4\right)
\]

(2)

![Table 1](image)

**Table 1** Chemical composition of Hastelloy X in wt%.

<table>
<thead>
<tr>
<th>Element</th>
<th>Ni</th>
<th>Cr</th>
<th>Fe</th>
<th>Mo</th>
<th>W</th>
<th>Mn</th>
<th>Co</th>
<th>Si</th>
<th>P,S</th>
</tr>
</thead>
<tbody>
<tr>
<td>wt%</td>
<td>47.2</td>
<td>22.8</td>
<td>18.5</td>
<td>8.2</td>
<td>0.71</td>
<td>0.49</td>
<td>0.73</td>
<td>0.24</td>
<td>&lt;0.008</td>
</tr>
</tbody>
</table>

![Fig. 1](image)  
Fig. 1. The compact tension specimen geometry.
where: $x = a/W$; expression valid for $a/W \geq 0.2$ [14].

For EPFM mode, the specimen’s uncracked ligament in CT should be smaller than Eq. (1).

Using Eqs. (1) and (2), forces were determined for LEFM and EPFM modes for each temperature.

Incremental polynomial method as a recommended data reduction technique was used for computing $\frac{da}{dN}$ (ASTM E 647-APPENDIX X1). In addition of $D$ and $\frac{da}{dN}$, $J$-integral was employed for calculating crack growth rate on the basis of energy models specially for EPFM modes. $J$-integral calculation for CT specimens was done according to ASTM E 1820 [15] as follows:

$$\begin{align*}
J &= J_e + J_p \\
J_e &= \frac{K^2(1 - \nu^2)}{E} \\
J_p &= \frac{\eta A_{pl}}{B_b b_0} \\
\end{align*}$$

where: $A_{pl}$ is the area under load-displacement, $B_b$ is the net thickness, $b_0$ = uncracked ligament and $\eta = 2 + 0.522 \frac{b_0}{W}$.

For operational definition of $\Delta J_{total}$, integration area during the specific load cycle was used which is shown in Fig. 3 [16].

2.2.1. Fatigue at room temperature

Fatigue tests performed at room temperature in both LEFM and EPFM modes. Table 2 shows the conditions of samples and forces used.

2.2.2. Induction heating TMF

TMF test were performed in both LEFM and EPFM modes. Used TMF cycles at constant temperatures, for in-phase, IP, and out of phase, OP, are shown in Fig. 4. The conditions of these tests are also presented in Table 3.

3. Results and discussion

3.1. Room temperature results

Results of crack growth rates were evaluated separately in LEFM and EPFM modes. Fig. 5 shows the results of crack growth rates versus stress intensity factor in the logarithmic scales. These results were extracted from three samples in LEFM mode and plane
stress mode for maximum forces of $F_{\text{max}} = 800, 700$ and 600 N. m and $C$, in Paris and Erdogan [18] equation were calculated by the least square analysis for LEFM samples at room temperature as follows:

$$\frac{da}{dN} = 10^{-11.8} \cdot A K^{5.7}$$ (6.a)  
$$\frac{da}{dN} = 3 \cdot 10^{-7} \cdot AJ^3$$ (6.b)  

For investigating EPFM, cyclic loads were used and the results were evaluated for stable and un-stable crack growth. Fig. 6 shows the maximum cyclic loads, i.e. $F_{\text{max}} = 8000$ and 6000 N used in EPFM. The arrows in Fig. 6 show starting of un-stable crack growth. As it mentioned for EPFM for intermediate and large scale forces using J-integral approach was more suitable than stress intensity factor. In the new approach used in this study for fatigue life prediction, J Calculation was initially used for the Resistance Curve of un-stable crack. Then the fatigue life was predicted from these curves. For the CT specimen at a point corresponding to crack length, a $(i)$, displacement, $v(i)$, and force, $P(i)$, on the graph of specimen load versus displacement, J-integral R-curve was calculated according to technique described in Ref. [15] as follows:

$$J_0 = \frac{K_{IC}^2}{E}(1 - \nu^2) + J_{pl(i)}$$  

$$J_{pl(i)} = J_{pl(i-1)} + \frac{\eta_{i-1}}{b_{i-1}} \left[ \frac{A_{pl(i)} - A_{pl(i-1)}}{b_{i-1}} \right] \left[ 1 - \gamma_{i-1} \frac{a_{i-1}}{b_{i-1}} \right]$$ (8)  

where: $K_{IC}$ can be calculated from Eq. (2) and

For investigating EPFM, cyclic loads were used and the results were evaluated for stable and un-stable crack growth. Fig. 6 shows the maximum cyclic loads, i.e. $F_{\text{max}} = 8000$ and 6000 N used in EPFM. The arrows in Fig. 6 show starting of un-stable crack growth. As it mentioned for EPFM for intermediate and large scale forces using J-integral approach was more suitable than stress intensity factor. In the new approach used in this study for fatigue life prediction, J Calculation was initially used for the Resistance Curve of un-stable crack. Then the fatigue life was predicted from these curves. For the CT specimen at a point corresponding to crack length, a $(i)$, displacement, $v(i)$, and force, $P(i)$, on the graph of specimen load versus displacement, J-integral R-curve was calculated according to technique described in Ref. [15] as follows:  

Table 2  
Test conditions of CT fatigue samples at room temperature.

<table>
<thead>
<tr>
<th>$a_0$ (crack length mm)</th>
<th>$F_{\text{max}}$ (N) ~</th>
<th>$F_{\text{max}}$</th>
<th>Fatigue fracture mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 mm (notch + 2 mm pre crack)</td>
<td>7000</td>
<td>0</td>
<td>EPFM</td>
</tr>
<tr>
<td>10 mm (notch + 2 mm pre crack)</td>
<td>8000</td>
<td>0</td>
<td>EPFM</td>
</tr>
<tr>
<td>9 mm (notch + 1 mm pre crack)</td>
<td>8000</td>
<td>0</td>
<td>EPFM</td>
</tr>
<tr>
<td>14 mm (notch + 6 mm pre crack)</td>
<td>6000</td>
<td>0</td>
<td>EPFM</td>
</tr>
<tr>
<td>14 mm (notch + 6 mm pre crack)</td>
<td>4000</td>
<td>0</td>
<td>EPFM</td>
</tr>
<tr>
<td>13 mm (notch + 5 mm pre crack)</td>
<td>2300</td>
<td>0</td>
<td>LEFM</td>
</tr>
<tr>
<td>15 mm (notch + 7 mm pre crack)</td>
<td>5000</td>
<td>0</td>
<td>EPFM</td>
</tr>
<tr>
<td>24 mm (notch + 16 mm pre crack)</td>
<td>1300</td>
<td>0</td>
<td>LEFM</td>
</tr>
<tr>
<td>24 mm (notch + 16 mm pre crack)</td>
<td>2300</td>
<td>0</td>
<td>EPFM</td>
</tr>
<tr>
<td>27 mm (notch + 19 mm pre crack)</td>
<td>800</td>
<td>0</td>
<td>LEFM</td>
</tr>
<tr>
<td>27 mm (notch + 19 mm pre crack)</td>
<td>700</td>
<td>0</td>
<td>LEFM</td>
</tr>
<tr>
<td>27 mm (notch + 19 mm pre crack)</td>
<td>600</td>
<td>0</td>
<td>LEFM</td>
</tr>
</tbody>
</table>

Fig. 4. In-phase and out-of-phase TMF cycles [17].
by equating Eq. (6.b) to Eq. (6.a) or Eq. (12) which will be described as an integrated equation. So, by cutting of the mentioned curve with R-curve of un-stable crack, \( D_{j\text{critical}} \) and \( D_{a\text{critical}} \) can be extracted. Also increase in \( D_j \) in the specified cycles should be clearly established. This information let us to predict the number of cycles lead to fatigue failure. For example, in a LEFM fatigue case for, \( F_{\text{max}} = 2300 \text{ N} \) and \( a_0 = 13 \text{ mm} \), which is shown in Fig. 7, it is possible to calculate the \( D_j - D_a \) equation. Following the above mentioned procedure, number of cycles to start the un-stable crack was calculated as follows:

The equation of fatigue case for, \( F_{\text{max}} = 2300 \text{ N} \), \( a_0 = 13 \text{ mm} \) is:

\[
D_j = 0.0054 \Delta a^4 - 0.1192 \Delta a^3 + 0.9078 \Delta a^2 - 0.4323 \Delta a + 12.92
\]  

(9)

and un-stable crack R-curve (see Fig. 7) can be determined by:

\[
\Delta j = -2.1514 \Delta a^2 + 48.187 \Delta a + 100.3
\]  

(10)

By equating Eq. (9) to Eq. (10), \( \Delta j_{\text{critical}} \) & \( \Delta a_{\text{critical}} \) were calculated, i.e. \( \Delta j_{\text{critical}} = 231 \text{ mj/mm}^2 \) and \( \Delta a_{\text{critical}} = 19.23 \text{ mm} \). The number of cycles to start the un-stable crack, \( N_j \) was calculated as 5274 cycles. The accuracy of the above method was evaluated by experimental validation test which conformed that the actual \( N_j \) was 5473 cycles which is in good agreement with the calculated one.

Results obtained for crack growth rate for hastelloy X- CT samples in EPFM, for intermediate and large stresses under plane stress mode could be integrated in one curve. Fig. 8 shows \( da/dN \) versus total J-integral for all the data presented in Table 2. Eqs. (6.b) and (11) are considered as suitable related equations for LEFM and EPFM respectively. The polynominal curve in Fig. 8 shows the best fitting of fatigue results in EPFM (for stable and un-stable cracks) at room temperature.

\[
\frac{da}{dN} = -3 \times 10^{-13} j^5 + 3 \times 10^{-10} j^4 - 1 \times 10^{-7} j^3 + 2 \times 10^{-5} j^2 + 0.00006 \cdot j 
\]  

(11)

The integrated results can be evaluated by stress intensity factor for both LEFM and EPFM. Fig. 9 shows the logarithmic curve of \( da/dN \) versus \( \Delta K \) based on the following related equation.

\[
\frac{da}{dN} = 10^{-9.8456} \cdot \Delta K^{3.699}
\]  

(12)

3.2. Results of TMF

TMF tests in LEFM mode was performed on the CT samples at constant temperature, for in phase and out of phase states. It should be noticed that \( \sigma_{ys} \) in Eq. (1) decrease with increasing temperature. In order to meet the requirements of Eq. (1) for satisfying LEFM, \( \sigma_{ys} \) in the maximum temperature during TMF must be used for calculating the maximum force.

TMF results at constant temperature (500 °C, \( F_{\text{max}} = 1500 \text{ N} \)) is shown in Fig. 10a. The related Paris-Erdogan equation (Eq. (13)) for LEFM which was extracted by least square analysis is shown in Fig. 10b. The amount of deviation associated with this equation is much more than that of Eq. (12) at room temperature.

\[
\frac{da}{dN} = 10^{-32.8} \Delta K^{20.4}
\]  

(13)

Comparing Eqs. (13) and (12), it is clear that for the same \( \Delta K \), crack growth rate at 500 °C is more than that of room temperature.

### Table 3

Test conditions of CT - TMF samples at various temperatures.

<table>
<thead>
<tr>
<th>( a_0 ) (crack length mm)</th>
<th>( F_{\text{max}} ) (N)</th>
<th>( F_{\text{min}} ) (N)</th>
<th>TMF phase</th>
<th>Temperature (°C)</th>
<th>Fatigue fracture mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1500</td>
<td>0</td>
<td>T-constant</td>
<td>500</td>
<td>LEFM</td>
</tr>
<tr>
<td>17.1</td>
<td>1500</td>
<td>0</td>
<td>IP</td>
<td>80-500</td>
<td>LEFM</td>
</tr>
<tr>
<td>17.2</td>
<td>1500</td>
<td>0</td>
<td>IP</td>
<td>415-500</td>
<td>LEFM</td>
</tr>
<tr>
<td>17.3</td>
<td>1500</td>
<td>0</td>
<td>OP</td>
<td>250-500</td>
<td>LEFM</td>
</tr>
<tr>
<td>17.4</td>
<td>1500</td>
<td>0</td>
<td>OP</td>
<td>740-600</td>
<td>LEFM</td>
</tr>
<tr>
<td>17.5</td>
<td>2600</td>
<td>0</td>
<td>T-constant</td>
<td>500</td>
<td>EPFM</td>
</tr>
<tr>
<td>17.6</td>
<td>3200</td>
<td>0</td>
<td>T-constant</td>
<td>500</td>
<td>EPFM</td>
</tr>
<tr>
<td>19.4</td>
<td>2600</td>
<td>0</td>
<td>IP</td>
<td>415-500</td>
<td>EPFM</td>
</tr>
<tr>
<td>19.5</td>
<td>2600</td>
<td>0</td>
<td>OP</td>
<td>370-500</td>
<td>EPFM</td>
</tr>
<tr>
<td>20</td>
<td>2600</td>
<td>0</td>
<td>T-constant</td>
<td>740</td>
<td>EPFM</td>
</tr>
</tbody>
</table>

Fig. 5. Log(da/dN) vs Log(\( \Delta K \)) for Hastelloy X at room temperature and LEFM.
Fig. 6. Cyclic load-displacement curve of CT samples in EPFM-large scale forces: (a) 8000 N, (b) 6000 N.

Fig. 7. J-integral resistance curve for un-stable crack and incremental increase in energy during of fatigue in EPFM and LEFM.
For example if one considers $\Delta K = 28$ MPa$\sqrt{m}$, the crack growth rates at the room temperature and $500^\circ C$ become 0.00025 and 0.0005 mm/cycle, respectively.

Results of TMF-IP and TMF-OP in LEFM are shown in Fig. 11. In order to compare of LEFM-TMF results based on energy, total J-integral of TMF and the related crack growth rates are calculated and presented in Table 4. It should be noticed that the total J-integral cannot predict crack growth rate and the TMF types (IP & OP), $\Delta T$ and $T_{\text{max}}$ have significant effects on fatigue life. Thus the instantaneous J-integral at each temperature should be used in the proposed model presented in Section 3.3.

TMF results in EPFM mode for intermediate and large scale stresses were investigated for the CT samples at constant temperature under IP and OP conditions. Fig. 12 shows cyclic
load–displacement and crack growth rate for intermediate stress for EPFM. Crack growth rate at \( T = 500^\circ C \) (Fig. 12d) was calculated by the following model.

\[
\frac{da}{dN} = 10^{-12.5} \cdot J^{2.7}
\]  

J-integrals calculated for TMF-EPFM are shown in Table 5. The high values of \( C \) and \( m \) in Eqs. (13) and (14) is an indication that at high temperatures, the fatigue results for Hastelloy X associated with large amount of error, which shows itself in the form of larger misfit in drawing of \( da/dN \) versus \( J \), see Fig. 12d. When J-integral is used for calculating the crack growth rate, therefore for investigating TMF, one should use another technique.

### Table 4
Total J-integral for TMF-LEFM results.

<table>
<thead>
<tr>
<th>TMF conditions</th>
<th>( T = 500^\circ C )</th>
<th>( T = 80-500^\circ C )</th>
<th>( T = 415-500^\circ C )</th>
<th>( T = 250-500^\circ C )</th>
<th>( T = 500-250^\circ C )</th>
<th>( T = 600-740^\circ C )</th>
<th>( T = 740-600^\circ C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta K ) (MPa( \sqrt{m} ))</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>Total J-integral (mj/mm²)</td>
<td>10.65</td>
<td>12.1</td>
<td>9.2</td>
<td>10.9</td>
<td>9.4</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>Experimental crack growth rate (mm/cycle)</td>
<td>0.0005</td>
<td>0.0004</td>
<td>0.0002</td>
<td>0.0004</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>Predicted crack growth rate by the proposed model (3.3) (mm/cycle)- without induced eddy current healing</td>
<td>0.00048</td>
<td>0.00041</td>
<td>0.00021</td>
<td>0.00043</td>
<td>0.00047</td>
<td>0.000062</td>
<td>0.00050</td>
</tr>
<tr>
<td>Predicted crack healing by induced eddy current (3.4) (mm/cycle)</td>
<td>~0</td>
<td>0.00019</td>
<td>0.00017</td>
<td>0.00025</td>
<td>0.00003</td>
<td>0.00026</td>
<td>0.00027</td>
</tr>
</tbody>
</table>

### 3.3. Proposed integration model for TMF study

As it mentioned in the previous section application of LEFM or EPFM for investigating under conditions of TMF, in phase and out of phase loading result in different crack growth rate and thus different thermo-mechanical fatigue life based on the crack growth rate will be estimated. Regarding the Classification of the TMF life behavior of engineering materials by Nitta and Kuwabara [19,20], Fig. 13, the TMF behavior of the samples in the current study under in plane stress condition has been classified as Type E’ up to 600 °C for more temperatures, it was classified as Type I.

For using TMF model while considering parameters such as material, loading, geometry and environmental factors, it was
assumed that the overall crack growth rate is due to two sorts of contributions, one is owing to pure fatigue and the other due to environmental effects. The environmental effects for the current study are the amounts of temperature (related to creep) and oxidation. For simplicity, a linear superposition law was applied [20]:

\[
\frac{da}{dN} (TMF) = \frac{da}{dN} (fatigue) + \frac{da}{dN} (creep) + \frac{da}{dN} (oxidation)
\]  

(15)

Worth mentioning that a Hastelloy X superalloy has outstanding oxidation resistance at temperatures up to 1200 °C. So in this study the samples did not expose to oxidation damages and the last term of Eq. (15) can be ignored. In conventional standard test methods, creep crack growth rates in metals at elevated temperatures using CT specimens subjected to static or quasi-static loading conditions, determination of steady-state creep crack growth rate is required. Because of temperature and stress variations of TMF (IP or OP), the conventional ways to calculate the crack growth rate as a result of creep \( \frac{da}{dt} \) cannot be used. Therefore in this study an energy-based approach was used, so that time dependence of the total J-integral for TMF for the same forces and temperatures were related to creep equivalent \( \frac{C}{C} \) of the material as the following:

\[
\frac{C}{C}(t) = \Gamma \int \left( W'(t) dy - T \frac{\partial u}{\partial x} ds \right)
\]  

(17)

where: \( W'(t) \) is instantaneous stress-power or energy rate per unit volume, \( \Gamma \) is path of the integral, that encloses the crack tip, \( ds \) is increment in the contour path, \( T \) is outward traction vector on \( ds \), \( u \) is displacement rate vector at \( ds \), \( T \). \( \frac{\partial u}{\partial x} \) \( ds \) is the rate of stress-power input into the area enclosed by \( \Gamma \). The magnitude of the \( \frac{C}{C}(t) \)-integral for the CT specimen at each point can be determined according to the following equation [21,22].

\[
\frac{C}{C}(t) = \frac{PV_e}{B(W-a)n+1} \left( 2 + 0.522 \frac{W-a}{W} \right)
\]  

(18)

where, \( n \) is creep exponent in the relationship between minimum creep rate and applied stress. The value of \( n \) may be obtained from creep test data. If creep tests cannot be performed, the accepted value of \( n \) from the literature may be used provided it is for the same type of materials [22]. \( V_e \) can be calculated by the following equations [23,24].

\[
V_e = \frac{a \dot{a} B}{E} \left[ \frac{2K^2}{E} \right]
\]  

(19)

where: \( \dot{\dot{V}} \) is load-line displacement rate, \( a \) is crack growth rate, \( da/dt \), \( P \) is applied force, \( E = E(1-\nu^2) \) for plane strain and \( E \) for plane stress, \( B \) is net section thickness and \( K \) is stress intensity factor.

Table 5

<table>
<thead>
<tr>
<th>J-integral calculated for TMF-EPFM results for ( \Delta K = 58 \text{ MPa} \sqrt{\text{m}} ).</th>
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<tr>
<td>J-integral (mj/mm²)</td>
</tr>
<tr>
<td>Experimental crack growth rate (mm/cycle)</td>
</tr>
<tr>
<td>Predicted rate by the proposed model (mm/cycle), without induced eddy current healing</td>
</tr>
<tr>
<td>Predicted crack healing by induced eddy current (mm/cycle)</td>
</tr>
</tbody>
</table>
Replacing \( V_c \) from Eq. (19) in Eq. (18), creep crack growth rate can be calculated.

\[
\dot{a} = \frac{E}{B \cdot 2 \cdot K^2} \left[ PV - \frac{C \cdot (n + 1)}{n} \right] \quad \text{(20)}
\]

where \( Y = (1/B(W-a))(2 + 0.522(W-a)/W) \).

According to Eq. (20), the creep crack growth rate is related to some parameters which they are themselves depended to temperature. Therefore because of temperature variation in TMF, \( \dot{a} \) can be calculated by the following relationship.

\[
\dot{a} \text{ creep in TMF} \left( \frac{mm}{cycle} \right) = \int_{T_1}^{T_2} \int_{P_1}^{P_2} \dot{a}_{\text{creep}}(T) m_c \cdot dT \quad \text{(21)}
\]

where \( m_c = dt/dT \), \( t \) is time in hour. For a TMF cycle with linear changing in temperature, \( m_c \) is equal to \( (T_2-t_1)/(T_2-T_1) \).

Eq. (21) defines the cyclic creep crack growth rate for TMF tests proposed in this study. For Hastelloy X super alloy as a creep resistance material, the magnitude of \( C^c \) (by Eq. (16)) and \( V \) at TMF testing temperatures have very small values and measurement of these factors requires hundred hours of precision testing. So in this study a combination of NSW model [25,21] for creep crack growth rate which is a function of \( C^c \) and strip yield model [26] was used to estimate crack growth rate as the following:

\[
\dot{a}_{\text{creep}}(T) = \left( \frac{n + 1}{C^c} \right) \left( \frac{1}{G^e A} \right) \left( \frac{R}{T_0} \right)^{(n+1)/n} \text{ for plane stress} \quad \text{(22)}
\]

where \( A \) is the coefficient from the secondary creep power law, \( G \) is the shear modulus, \( r_c \) is the radius of the creep plastic zone (assumed to be equal to \( \rho \), the plastic zone size calculated by Dugdale strip yield model [27], and is equal to \( \rho = \pi K^2/8 \sigma_0^2 \)), \( \epsilon_f^c \) is the creep ductility (\( \epsilon_f^c = \epsilon_f \) for plane stress and \( \epsilon_f^c = \epsilon_f/50 \) for plane strain) and \( I_N \) is a non-dimensional function of the plastic strain hardening exponent \( N \). \( I_N \) was determined numerically in [28] as the following:

\[
I_N = 7.2 \left( 0.12 + \frac{1}{N} \right)^{0.5} - \frac{2.9}{N} \quad \text{for plane stress} \quad \text{(23.a)}
\]

\[
I_N = 10.3 \left( 0.13 + \frac{1}{N} \right)^{0.5} - \frac{4.6}{N} \quad \text{for plane strain} \quad \text{(23.b)}
\]

Also \( V_c \) is approximated by \( V_c = V_c = A(t)^{10} [26] \) and \( C^c \) could be calculated by Eq. (18).

For the current study, the relation of the mentioned material properties, \( E \), \( n \) and \( N \), to temperature were obtained from [1,29] and the results are given in Table 6.

The new proposed model in the current study, for calculation of the pure fatigue crack growth rate at various temperature, is in fact an energy based equation at room temperature (e.g. Eqs. (6.b) and (11)) which can be used in association with yield stress and the crack tip opening displacement at any temperature which TMF test is performed. The relationship between \( J \) and CTOD has been established by Shih [30] as the following:

<table>
<thead>
<tr>
<th>( \dot{a} \text{ creep in TMF} \left( \frac{mm}{cycle} \right) = \int_{T_1}^{T_2} \int_{P_1}^{P_2} \dot{a}_{\text{creep}}(T) m_c \cdot dT )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{a}_{\text{creep}}(T) = \left( \frac{n + 1}{C^c} \right) \left( \frac{1}{G^e A} \right) \left( \frac{R}{T_0} \right)^{(n+1)/n} \text{ for plane stress} )</td>
</tr>
</tbody>
</table>

**Table 6** Relationship of the required parameters for calculation of the creep crack growth rate, with temperature for Hastelloy X.

<table>
<thead>
<tr>
<th>( E ) (GPa)</th>
<th>( n ) - ( T ) (°C)</th>
<th>( n ) - ( T ) (°C)</th>
<th>( N ) - ( T ) (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E = 0.0653T + 210.39 )</td>
<td>( n = 3E \cdot 08 T^3 + 4E \cdot 05 T^2 - 0.0002T )</td>
<td>( N = 7E \cdot 10^4 T^3 + 6E \cdot 07 T^2 + 1E \cdot 05 T + 0.1502 )</td>
<td></td>
</tr>
</tbody>
</table>
CTOD = \frac{J}{\sigma_0} \quad (24)

where \(\sigma_0\) is a dimensionless constant given in [16] and ASTM E 1820-11, the value of the \(\sigma_0\) depends on the hardening behavior of the material. For this study, it was found that Eq. (24) follows the dugdale model [27] for strip yield model and so \(\sigma_0\) is equal to one. The load- line displacement, \(V\), for CT samples can be given by the following equation [15].

\[ CTOD = \delta_i = \frac{K_i^2}{2\sigma_0 E} + \frac{r_p (w - a_0) V_{pl}}{r_p (w - a_0) + a_0 + z} \quad (25) \]

where \(r_p\) is the plastic rotation factor, its values have been given in Ref. [15], \(r_p\) is consider to be 0.521 and \(z\) is equal to 10 mm for the current study of TMF tests.

Combining Eqs. (25) and (24) and inserting the value of \(J(\sigma_0, V_{pl, fatigue})\) in the Eqs.(6.b) and (11), the pure fatigue equation for crack growth rate, \(a_{fatigue}\) can be calculated for each temperature. Because of temperature variation in TMF, \(a_{fatigue}\) can be calculated by the following relationship.

\[ a_{fatigue \text{ in TMF}} \text{ (mm/cycle)} = \int_{T_{lap1}}^{T_{lap2}} f(T) \cdot m_f \cdot dT \quad (26) \]

where \(f(T)\) is the modified pure fatigue crack growth rate (mm/cycle) for TMF which is calculated by substitution of \(J(\sigma_0(T), V_{pl, fatigue}(T))\) in the room temperature equation (e.g. Eqs. (6.b) and (11)) and \(m_f = \frac{dN}{dT}\) which is equal to \(1/(T_2-T_1)\) for a TMF cycle having a linear change in temperature.

It should be mentioned that for Hastelloy X super alloy, the creep crack growth rate has very small value in comparison with that of fatigue, up to 600 °C. Therefore creep crack growth could be ignored for temperatures up to 600 °C and as a result creep/fatigue interaction was considered as negligible in this research. The relationship between the yield stress and temperature obtained from [29] for Hastelloy X and also the relation between \(V\) and temperature was extracted for each experimental TMF tests and used to calculate the pure fatigue crack growth rate. For calculating various parameters of the proposed model, MAPLE 15 was used [31].

R-curves for TMF under large scale stress where drawn for two temperatures. Fig. 14 shows the R-curves of un-stable cracks at 500 and 740 °C. Then, using the results of TMF-J integral versus \(Aa\) and cutting of the TMF-R-curve of un-stable crack, \(A_{Jcritical}\) and \(A_{a critical}\) for TMF tests were extracted and used for the TMF life prediction.

It is said [32] that creep/fatigue interaction can produce a complex process of damage, involving fatigue and creep damages due to cyclic stress at high temperature. Also Tang proposed a dual
scale fatigue crack growth model [33] to analyze the thermal-mechanical effects in materials under effects of creep and cyclic loading. In the current study, interaction between creep and fatigue was observed for temperatures between 600 °C and 740 °C for TMF-OP which was shown in Fig. 15. This figure shows crack growth rate increased with decreasing force from 1652 N to 620 N and increasing temperature from 600 °C to 695 °C linearly. This indicates that the combined effect of force (which its reduction should reduces crack growth rate) and temperature (which by its increase, the flow of materials would increase caused an increase in the crack growth rate. In the other words, the interaction between the generated dislocations with those already existed within the material, as well as the interaction between the dislocation and the precipitates can cause localized strengthening of the material, hence increased in the crack growth rate was much further than the amount of reduction in growth rate due to force reduction) up to 695 °C. With further increase of temperature to 740 °C and decrease in force to about 100 N the rate of crack growth sharply decreased. This can be due to further softening of the material due to release of dislocations from barriers which helped the material to flow rather than helping the growth of the crack. 

A major part of the creep/fatigue interaction which was considered in the cyclic creep part of the proposed model was explained in Eqs. (18)–(23). Certainly other factors such as precipitates can affect the crack growth rate which was not considered in this research.

It should be noted that the proposed model for TMF was based on damage mechanisms contributions for fatigue and creep. Thus the linear superposition law, Eq. (15), considers the accumulation of damages due to plastic strains on the crack tip for the fatigue part and time dependent damages such as micro void accumulation for the creep part. The fatigue crack growth part, $da/dN_{fatigue}$ was modeled for cyclic temperatures and the creep crack growth part, $da/dN_{creep}$, was modeled for cyclic stresses, and the interactions of the fatigue/creep are considered separately in Eq. (15).

Comparing the actual experimental data with data obtained from the proposed model (see Tables 4 and 5), it was demonstrated that the actual crack growth rates are less than those predicted by the proposed model. Also the low values of experimental crack growth rates at TMF-IP & OP at max temperature 740 °C, even in the presence of creep, shows the effect of another important parameter which is named as induced eddy current healing in this study. This parameter will be discussed in the following section.  

3.4. TMF crack growth healing by induced eddy current

For investigating crack healing-crack closure during induction heating, the mechanism of electromagnetic induction heating was evaluated. For calculating an electromagnetic field it is necessary to solve Maxwell’s equations as it has been indicated in [34–36]. This procedure is as follow:

$$
V \times B = \mu j \quad \text{(Ampere’s law)} 
$$

$$
V \times E = -\frac{\partial B}{\partial t} \quad \text{(Faraday’s law)} 
$$

$$
\mathbf{j} = \sigma \mathbf{E} \quad \text{(Ohm’s law)} 
$$

where $E$ is the electric field intensity, $B$ is the magnetic flux density, $\mathbf{j}$ is the current density, $\mu$ is the magnetic permeability and $\sigma$ is the electrical conductivity of the medium.

From the Maxwell’s Equation: div $B = 0$. In the scalar field, $\varphi$ could be defined by a two dimensional vector curling operator as [37,38]:

$$
B = \text{curl} \varphi = V \times \varphi = \left( \frac{\partial \varphi}{\partial x_2} - \frac{\partial \varphi}{\partial x_1} \right) 
$$

$$
-\text{div} \left( \frac{1}{\mu} \mathbf{V} \varphi \right) = \text{curl} \left( \frac{1}{\mu} \text{curl} \varphi \right) = \mathbf{j} 
$$

After calculating the induced eddy current density by Eqs. (27)–(31), the thermal problem should be solved in the temperature field resulting from the joule heat according to [38]:

$$
C(T) \frac{\partial T}{\partial t} - \text{div} \left( \lambda(T) \mathbf{V} T \right) = P 
$$

$$
P(x, t) = \frac{(j \mu x e^{j \omega t})^2}{\sigma(T(x, t))} 
$$

where $C$ is the specific heat, $\lambda$ is the thermal conductivity and $P$ is the joule power.

The final model consists of induced eddy currents and temperature distribution which can be solved by Eqs. (27)–(33). It has been demonstrated [12] that the eddy current flows around the crack tip; it accumulates in the crack tip area, leading to much higher current density in the crack tips than in other places in the vicinity of the crack. The higher current density produces more
heat that results in a significant rise in temperature. Since the thermal expansion rate of metallic material is proportional to the temperature, the thermal expansion in the vicinity of the crack tip will be much more than the thermal expansion in areas far from the crack tip, thus the thermal expansion of the crack tip will be suppressed by the surrounding area. Therefore, there is thermal stress around the crack tip due to the compressive force which is exerted by the surrounding area [12].

The theoretical compressive thermal stress due to change of temperature rise and thermal expansion can be calculated by Hooke’s law:

\[
\sigma = E(T) \cdot \alpha(T) \cdot \Delta T
\]  

(34)

where \( \alpha \) is the thermal expansion coefficient of the specimen and \( \Delta T \) is the temperature difference between the crack tip area and the neighboring area of the fatigue crack.

Fig. 16 shows the eddy current density, temperature distribution and compressive stress in the crack tip for induction heating TMF tests. The Quick field software according to Ref. [39] can be used for simulation of induction heating based on the Eqs. (27)–(33). The results were validated by experimental measurement of temperatures by K-type thermocouple and also IR-temperature measurements. The compressive stresses led to decrease in crack growth rate and so it plays a role as a crack closure effect and so the Eq. (15) should be modified as the following:

\[
\frac{da}{dN}^{(TMF)} = \frac{da}{dN}^{(fatigue)} + \frac{da}{dN}^{(creep)} + \frac{da}{dN}^{(oxidation)} - \frac{da}{dN}^{(eddy current healing)}
\]  

(35)

It should be mentioned that the compressive stresses have different effects on the pure fatigue and creep contributions. For calculation of the last terms in the Eq. (35), the effects of compressive stresses on fatigue and creep should be measured by Eqs. (24)–(26). By substituting the compressive stresses in Eq. (24) at each temperature one can measures the effect of stress on fatigue. For creep, Eqs. (21)–(23) should be used by substituting the compressive stress in the relationship between \( V_c \) and stress, i.e. \( V_c \approx \dot{\epsilon}_c = A\sigma^n \).

The predicted crack healing by induced eddy currents were calculated by the above approach and the results are given in Tables 4 and 5. For this research, maximum healing effects was found to be due to creep contribution of the TMF-LEFM with maximum temperature 740 °C.

The robustness of the presented model was checked. For this purpose contribution of fatigue and creep on crack growth rate model were calculated by MAPLE software [31]. However the effect of induction heating on crack growth model was established by FEM analysis using Quick field software [39]. The root relative squared error between the calculated crack growth rates obtained

Fig. 16. [a] Temperature distribution, [b] eddy current streams and compressive stress on the crack tip [12] with 17 mm length, [c] temperature gradient on the crack tip at three induction times and [d] compressive stress on the crack tip versus maximum temperature.
via Eqs. (15)-(26) and those obtained experimentally were determined by Eq. (36) below. When n is the number of TMF tests.

\[
\%\text{RRE} = \sqrt{\frac{\sum_n \left( \frac{dN}{dn} (\text{experimental TMF}) - \left( \frac{dN}{dn} (\text{calculated TMF by model}) - \frac{dN}{dn} (\text{crack healing}) \right) \right)^2}{\sum_n \left( \frac{dN}{dn} (\text{experimental TMF}) - \left( \frac{dN}{dn} (\text{average of experimental TMF}) \right) \right)^2}} \times 100
\]  

(36)

Using the results presented in Tables 4 and 5 and Eq. (36), the mentioned error for n tests was % 4.3, which was included all experimental error sources and uncertainly in the proposed model.

Worth mentioning that the effects of different parameters such as range of temperature, AT, range of stress intensity factor, AK (includes force and crack length) and maximum temperature of TMF testing on the relative errors of the proposed model were considered and used for error analysis. Results show that most errors of the model are due to TMF-OP which seems to be related to microstructure effects. For some other TMF tests, it was found that a combined parameter such as \( \phi = \frac{AT}{\text{max}} \) is a suitable variable to establish a relationship for relative errors (%RE) of the proposed model as follows:

\[
\%\text{RE} = -0.0186\phi^2 + 0.1564\phi + 2.0976
\]

(37)

As a brief review, the results of this research showed that crack growth on the base of experimental TMF and room temperature fatigue on the Hastelloy X superalloy can be extracted from Eqs. (6), (9)-(11), (13), (14). Also a developed crack growth model was proposed by Eqs. (16), (21), (26) and (35), using the combination of Eqs. (24) and (25) and calculating the crack healing in TMF (by substituting the thermal compressive stress in the related equations of fatigue and creep), this was one of the contributions of this research to the scientific world.

4. Conclusions

Based on the predicted and the experimental results obtained in this research for crack growth rates, the following conclusions were drawn.

1- By extrapolating the fatigue curve of \( \Delta N - \Delta a \) and cutting off the resistance curve for un-stable crack, \( \Delta a_{\text{critical}} \) and \( \Delta a_{\text{critical}} \) were extracted. Thus by this method fatigue life of a material can be predicted at room temperature and also at high temperatures.

2- The TMF behavior of Hastelloy X under plane stress condition classified in Type E' at temperatures up to 600 °C and for higher temperatures, it could be classified in Type I.

3- The new aspect of the proposed model is the use of room temperature equations for fatigue section of TMF crack growth rate. The temperature effects can be considered by fracture mechanic equations. So the cyclic creep part of TMF was investigated on the base of NSW and yield strip model.

4- This study showed that the TMF crack healing by induced eddy currents can be calculated in the AC magnetic and temperature fields. It was demonstrated that the eddy current flows around the crack tip; it accumulates in the crack tip area, leading to much higher current density in the crack tips than in other areas in the vicinity of the crack. The higher current density produced more heat and resulted to a significant rise in temperature. The compressive thermal stress due to change of thermal expansion resulted to crack healing.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version, at http://dx.doi.org/10.1016/j.ijfatigue.2016.12.036.

References


