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Mohammad Saidi Mehrabad; Mona Anvari

*Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

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Provident decision making by considering dynamic and fuzzy environment for FMS evaluation

Mohammad Saidi Mehrabad and Mona Anvari*

Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

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When evaluating complexity, cost and risk increase, it is difficult to make a proper decision. In such situations it is necessary to develop a model which simulates a decision maker’s mind and consider both a dynamic and a fuzzy environment. In this study future oriented indices are presented which enable us to consider the effect of future changes in the index value during the decision making process. These future oriented indices are named provident indices. Also in this study to effectively integrate these multiple criteria into the decision making process, based on the analysis of the decision situation in such assessments, a suitable concept is selected. This method is based on the preference ranking organisation method for enrichment evaluations (PROMETHEE) which brings together flexibility and simplicity for the user and is therefore chosen for the enhancement towards the evaluation of fuzzy data on preferences, scores and weights. The model developed to investigate these impacts cannot perfectly reproduce all the events of the real system, but it can consider a fair number of elements of variability, which should be identified and analysed by considering present conditions and prediction about criteria values in future periods. Such a model may provide solutions with a high degree of robustness and reliability, comparable with those obtained by direct experimentation, but with a low computational cost. The uniqueness of this paper lies in two important areas: first, the incorporation of variability fuzzy and provident measures in the performance of alternatives into the decision making process; and second, is in the application of fuzzy PROMETHEE that provides the decision maker with effective alternative choices by identifying significant differences among alternatives and appropriate choices through considered future periods, and presents graphic aids for better interpretation of results. A comprehensive numerical example of a flexible manufacturing system (FMS) is provided to illustrate the results of the analysis. In a real-world manufacturing environment, the dynamics of an FMS and its stochastic characteristics require a specific approach for evaluation. This paper specifically focuses on FMSs due to the complexities involved in their proper evaluation that include factors such as high operational and managerial expertise in system implementation phases, high costs and risks. Due to these, evaluation, justification, and implementation of an FMS have been areas of major concern and importance for practitioners and researchers. In this case, various strategic, economic and operational criteria that envelop quantitative, qualitative, tangible, and intangible factors are considered.

Keywords: provident decision making; fuzzy indices; PROMETHEE; FMS; performance analysis

*Corresponding author. Email: manvari@iust.ac.ir

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1. Introduction

Flexible manufacturing systems (FMSs) are envisioned as an effective solution for delivering products at low cost, high quality, high variety, and short lead times. They provide many important benefits such as greater manufacturing flexibility, reduced inventory, reduced floor space, faster response to shifts in market demand, lower lead times, and a longer useful life of equipment over successive generations of products (Kulak and Kahraman 2005). It is for this reason that manufacturers have increased investments in these new technologies. Due to these increased investments, evaluation, justification, and implementation of an FMS have been areas of major concern and importance for practitioners and researchers. Research in this area has matured from managerial and conceptual issues facing the justification of FMS to the development of analytical tools and models for evaluation and justification purposes. Evaluating alternative manufacturing technologies in the presence of multiple performance measures is often a difficult task for the decision maker (DM). It is for this reason that justification and evaluation of FMSs has been receiving significant attention in the manufacturing circles (Talluri et al. 2000). FMS adoption and justification literature has focused on the reasons for lack of broad implementation of these systems given their advantages and benefits.

The initial generation of models in this area included traditional cost-based capital budgeting approaches, such as return on investment which ignore time value of money, payback, return on investment (ROI) and various discounted cash flow models such as present worth, annual worth and internal rate of return. Application of traditional capital budgeting methods does not fully account for the benefits arising from intangible factors of an automated manufacturing system (AMS) (Kahraman et al. 2000). Economic methods alone will not be enough for justification of FMSs but investment evaluation indices play an important role in today’s competitive manufacturing environment. Shrinking profit margins and diversification require careful analysis of investments, and the decisions regarding these investments are crucial to the survival of the manufacturing firm (Karsak and Tolga 2001).

More specifically, the reasons for the failure to adopt these systems include factors such as high operational and managerial expertise in system implementation phases, high costs and risks associated with the systems, emphasis on short-term performance measures, and inappropriate costing approaches. Strategic justification models that incorporate a variety of factors help address some of these issues and concerns. These models incorporated intangible benefits along with strategic and tactical measures for justification purposes. Small and Chen discuss the results of a survey conducted in the US that investigated the use of justification approaches for AMS. According to their findings, manufacturing firms using hybrid strategies, which employ both economic and strategic justification techniques, attain significantly higher levels of success from advanced technology projects (Small and Chen 1997). Productivity, quality, flexibility and other intangibles should be examined in terms of potential returns through enhancement of long-term business competitiveness as well as in terms of a comprehensive evaluation of internal costs (Proctor and Canada 1992). In the research field a number of articles focus on integrating the qualitative and quantitative aspects to evaluate the benefits of AMS because certain criteria cannot be expressed in quantitative terms. When flexibility, risk and non-monetary benefits are expected, and particularly if the probability distributions can be subjectively estimated, analytical procedures may be used. Strategic justification methods are qualitative in
nature, and are concerned with issues such as technical importance, business objectives, competitive advantage, etc.

When strategic approaches are employed, the justification is made by considering long-term intangible benefits. Hence, using these techniques with economic or analytical methods which are short-term management issues would be more appropriate.

Data type as much as evaluation method would be important in justification of FMSs. For a long time, it has been recognised that an exact description of many real-life physical situations may be virtually impossible. This is due to the high degree of imprecision involved in real-world situations.

Zadeh (1965, 1968) in his seminal papers proposed fuzzy set theory as the means for quantifying the inherent fuzziness that is present in ill-posed problems. Fuzziness is a type of imprecision which may be associated with sets in which there are no sharp transition from membership to non-membership. Fuzzy logic is basically a multi-valued logic that allows intermediate values to be defined between conventional evaluations like yes/no, true/false, black/white, etc. As Zadeh (1965, 1968) puts it, human goals matter in the case of decision processes, and therefore a wide gap exists between theory and practice in decision analysis. The concept of fuzzy sets is a way to deal systematically with unsharp figures, which better represent the reality.

An effective way to express factors including flexibility, quality of the products, enhanced response to market demand, end reduction in inventory, which can neither be assessed by crisp values nor random processes, is using linguistic variables or fuzzy numbers (Karsak and Tolga 2001). The collection of accurate data on the production techniques and the industrial installations poses further difficulties: due to varying measurements and the differences in the input parameters, comparable exact data is rarely available. Therefore, a description of the techniques with the help of fuzzy numbers would seem to be more realistic than with crisp numbers. Only a flexible comprehensive assessment approach can foster the discussion of the political and industrial DMs on the most relevant aspects (Geldermann et al. 2000b). The fuzzy set theory appears as an important tool to provide a multi-criteria decision framework that incorporates the vagueness and imprecision inherent in the justification and selection of AMSs.

As with any research area of study, the development of the field provides more effective, insightful, and powerful tools for analysis of managerial problems. Recently, fuzzy multi-attribute decision-making techniques are applied to the acquisition of AMSs (Kulak and Kahraman 2005). Since it is difficult to provide a detailed review of most of the works in this area, in Table 1 we mainly highlight some important issues in the last two decades that face the FMS evaluation and justification problem. It can be seen that none of these studies considers the fuzzy concept and all kinds of indices together. This paper specifically focuses on FMS due to the complexities involved in their proper evaluation, which include the consideration of various strategic, economic and operational indices that envelop quantitative, qualitative, tangible, and intangible factors.

Also when evaluation complexity, cost and risk increase, it is necessary to develop a model which simulates the DM’s mind and consider both a dynamic and a fuzzy environment. In this study provident measures are presented which enable us to consider the effect of future changes in the measure value during the decision making process. These measures integrate the predicted values of a specified index for future considered periods with respect to their importance determined by the DM or product features such as product life-cycle (PLC).
Table 1. Some important issues in the last two decades that face the FMS evaluation and justification problem.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Economic</th>
<th>Strategic</th>
<th>Operational</th>
<th>Crisp</th>
<th>Fuzzy</th>
<th>Case</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myint and Tabucanon</td>
<td>1994</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td></td>
<td>Select the most appropriate machines for FMS</td>
<td>Analytical hierarchy process (AHP) and goal programming (GP)</td>
</tr>
<tr>
<td>Sheng and Sueyoshi</td>
<td>1995</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td></td>
<td>FMS evaluation and selection</td>
<td>Data envelopment analysis (DEA), AHP and simulation</td>
</tr>
<tr>
<td>Khouja</td>
<td>1995</td>
<td>#</td>
<td></td>
<td>#</td>
<td>#</td>
<td></td>
<td>Advanced manufacturing technology selection</td>
<td>DEA, multi attribute decision making (MADM)</td>
</tr>
<tr>
<td>Shafer and Bradford</td>
<td>1995</td>
<td>#</td>
<td></td>
<td>#</td>
<td>#</td>
<td></td>
<td>Selection of cellular manufacturing systems (CMSs)</td>
<td>Simulation and DEA</td>
</tr>
<tr>
<td>Sambasivarao and Deshmukh</td>
<td>1997</td>
<td>#</td>
<td></td>
<td>#</td>
<td>#</td>
<td></td>
<td>AMSs justification</td>
<td>Decision support system (DSS)</td>
</tr>
<tr>
<td>Baker and Talluri</td>
<td>1997</td>
<td>#</td>
<td></td>
<td>#</td>
<td>#</td>
<td></td>
<td>Technology selection</td>
<td>Cross-efficiency measures in DEA</td>
</tr>
<tr>
<td>Karsak</td>
<td>1998</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td></td>
<td>Robot selection</td>
<td>DEA, fuzzy robot and selection algorithm</td>
</tr>
<tr>
<td>Authors</td>
<td>Year</td>
<td>Category</td>
<td>Methodology</td>
<td>Main Focus</td>
<td></td>
<td></td>
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<td>-----------------------</td>
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<td>---------------------------------------------------------------------------</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perego and Rangone</td>
<td>1998</td>
<td># #</td>
<td>Assessment and selection of AMS</td>
<td>MADM techniques</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D’Angelo et al.</td>
<td>1998</td>
<td># #</td>
<td>Quantitative evaluation of the technological performance of an FMS with particular reference to the printed circuit board assembly (PCBA) sector</td>
<td>Response surface methodology (RSM)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Talluri et al.</td>
<td>2000</td>
<td># # #</td>
<td>Selection of FMSs</td>
<td>Simulation, DEA and nonparametric statistical procedures</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Karsak and Tolga</td>
<td>2001</td>
<td># #</td>
<td>Rank the AMS investment alternatives</td>
<td>Fuzzy decision algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beskese et al.</td>
<td>2004</td>
<td># #</td>
<td>Quantification of flexibility in AMSs</td>
<td>New model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kulak and Kahraman</td>
<td>2005</td>
<td># # #</td>
<td>Comparison of FMSs</td>
<td>Axiomatic design (AD)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Azadeh and Anvari</td>
<td>2006</td>
<td># #</td>
<td>Optimisation of operators allocation in CMS</td>
<td>Simulation DEA, principle component analysis (PCA), numerical taxonomy (NT)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Azadeh et al.</td>
<td>2007</td>
<td># #</td>
<td>Optimisation of operator allocation in CMS</td>
<td>Simulation fuzzy DEA</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
Triangular fuzzy numbers are used throughout the analysis to quantify the vagueness inherent in the financial estimates such as periodic cash flows, experts’ linguistic assessments for some of the strategic justification indices and the operational indices related to short-term management issues, such as percentage of tardy jobs which can be obtained from the simulation model.

To effectively integrate these multiple indices into the decision making process and the suitability of FMS investment alternatives requires the use of more advanced models such as fuzzy decision-making techniques. In this study, based on the analysis of the decision situation in FMS assessments, a suitable concept is selected. The method PROMETHEE (Brans et al. 1986) brings together flexibility and simplicity for the user and is therefore chosen for the enhancement towards the evaluation of fuzzy data on preferences, scores and weights.

The uniqueness of this paper lies in two important areas: the first is incorporation of variable fuzzy and provident measures in the performance of alternatives into the decision making process; and the second is in the application of fuzzy PROMETHEE and fuzzy C-Means procedures that provide the DM with effective alternative choices by identifying significant differences among alternatives and appropriate choices through considered future periods, and presents graphic aids and sensitivity analysis for better interpretation of results. This procedure allows for grouping technologies so that differences in the performance of the systems in a specific group are insignificant. This provides the decision maker with alternative choices within a group. Such a procedure will be of high value for the decision maker, since they can either base their final decision on a tie breaking factor when alternative technologies do not clearly differ or base the decision on other tangible and intangible measures that could not have been incorporated into the decision models.

The paper is organised as follows. Section 2 provides an introduction to fuzzy PROMETHEE. Sections 3 and 4 are dedicated to provident and fuzzy FMSs evaluation indices and an algorithm is proposed in Section 5 for assessing efficiency of FMS’s alternatives and sensitivity analysis. The methodology is illustrated through its application on a data set in Section 6. The final section of the paper offers conclusions and suggests areas for the future research.

2. Fuzzy PROMETHEE method

2.1 Select a proper method for FMS evaluation

Geldermann et al. (2000b) demonstrated that the outranking methods as a special subgroup of multi-criteria decision making (MCDM) methods are particularly suitable for integral decision making through the notion of weak preference and incomparability, which better represent the real decision situation (Spengler et al. 1998, Geldermann et al. 2000a).

In this study, based on the analysis of the decision situation in FMS assessments, a suitable concept is selected. The method PROMETHEE brings together flexibility and simplicity for the user and is therefore chosen for the enhancement towards the evaluation of fuzzy data on preferences, scores and weights (Brans et al. 1986). It can be stated that outranking via fuzzy PROMETHEE helps to gain an insight into the decision maker’s preference structure and to focus attention on critical issues, which could be regarded as the aim of soft decision analysis.
From an analytical point of view, the interpretation of the values on the nominal scale with verbal expression as a ratio scale using numbers is also doubted. The basic principle of preference measurement is the establishment of a value function based on a simple addition of scores representing goal achievement according to each index, multiplied by the particular weights. Therefore, the concept of trade-off between the scores on different criteria is central to the interpretation of the value function, as it also underlies the multi attribute value/utility theory (MAVT/MAUT). This means that complete ‘compensation’ between attributes is possible, so that a sufficiently large gain in a lesser attribute will eventually compensate for a small loss in a more important attribute, no matter how important one attribute is (Stewart 1992).

In an FMS, for example, good results concerning the floor space requirement might counterbalance high capital and operating cost, but this mathematical representation does not match the real conditions for decision making. In order to overcome the assumption of complete compensation and of the existence of a ‘true’ ranking of the alternatives which only needs to be discovered, the outranking methods have been developed. Outranking rather takes into account that preferences are not constant in time, are not unambiguous, and are not independent of the process of analysis (Roy and Bouyssou 1993).

Especially the principles of weak preference and incomparability are valuable in FMS decision support, because they better represent the real situation. The ELECTRE approaches (Roy and Bouyssou 1993) are a popular manner of rendering the outranking concept operational, but this method has a drawback through its complexity, stemming from the nuances in the comparisons: through the underlying assumptions for the algorithm, the method is rather difficult to explain to decision makers in industry, especially since the introduced thresholds do not have a realistic meaning (Brans et al. 1986, Maystre et al. 1994). To overcome these obstacles, the outranking method PROMETHEE has been developed, bringing together flexibility and simplicity for the user.

The fuzzy objective function, to which it mainly refers, is not always sufficient for soft decision analysis. More benefits can be gained by also evaluating fuzzy scores on the criteria. Therefore, the integration of fuzzy algebra into the PROMETHEE algorithm is applied in this paper. Since it is particularly difficult to realistically model the true-shape of the membership function, triangular fuzzy numbers are used. The advantage can be seen in their good practicability and ease of understanding.

Roy and Bouyssou (1993) found that the fuzzy notion of the preferences may not be sufficient for an adequate comprehensive decision support, but that the consideration of fuzzy scores and fuzzy weights might be useful in ill-structured decision situations. Therefore, this paper applies the enhancement of PROMETHEE towards fuzzy logic in order to consider not only fuzzy preferences, but simultaneously fuzzy scores and fuzzy weights. The applied method is based on a proposed method by Geldermann et al. (2000b).

2.2 Fuzzy PROMETHEE

Geldermann et al. (2000b) suggested the use of fuzzy algebra throughout the PROMETHEE algorithm. Since it is particularly difficult to realistically model the ‘true’ shape of the membership function, the use of the trapezoidal fuzzy numbers was proposed for the enhancement of PROMETHEE by Geldermann et al. (2000b) but Delgado et al. (1998) argued that any fuzzy number can be represented as a trapezoidal or
a ‘quasi-trapezoidal’ fuzzy number, having the same basic attributes as the original fuzzy number. In this paper the indices are defined in triangular fuzzy numbers and triangular fuzzy numbers are a specific case of the trapezoidal fuzzy interval with $m_l = m_u$. The membership function for the triangular fuzzy number can be mathematically formulated as:

$$
\mu(x) = \begin{cases} 
0, & \text{for } x \leq m - \alpha \text{ or } m + \beta \leq x \\
1 - \frac{m - x}{\alpha}, & \text{for } m - \alpha < x < m \\
1, & \text{for } x = m \\
1 - \frac{x - m}{\beta}, & \text{for } m < x \leq m + \beta 
\end{cases}
$$

(1)

where $\alpha$ and $\beta$ are the left and right spread of the triangular fuzzy and with the centre $m$ the number which belongs with certainty to the set of available values. This triangular fuzzy is represented by the notation $\tilde{M} = (m; \alpha; \beta)$.

In this study, a method based on Geldermann et al. (2000b) is proposed for triangular fuzzy numbers because of the considered form for the indices membership function.

**Fuzzy PROMETHEE algorithm:**

**Step 1:** Throughout the analysis, we assume that there are a group of $J$ decision-makers ($D_1, D_2, \ldots, D_J$), who assess the importance weights of $K$ indices and suitability of $n$ FMS alternatives under each of these $K$ index.

**Step 2:** Specify for each index $f_k$ a generalised preference function $p_k(d)$.

**Step 3:** Define a vector containing the fuzzy weights (which do not need to be normalised to $\sum_{t=1}^{K} \tilde{w} = 1$):

$$
\tilde{w}^T = [\tilde{w}_1; \ldots; \tilde{w}_K], \text{ with } \tilde{w}_t = (m_t, \alpha_t, \beta_t).
$$

It is assumed that the DMs are using a set of weights $W = \{VL, L, M, H, VH\}$, where: VL indicates ‘very low’; L, low; M, medium; H, high; and VH, very high, to show the importance of each index (Karsak and Tolga 2001). The membership functions for importance-related weights are depicted in Figure 1.

Suppose that $w_{ij}$ is the linguistic weight given to economic criteria $C_1, C_2, \ldots, C_E$, strategic justification criteria $C_{E+1}, \ldots, C_{E+S}$, and operational criteria (obtained from simulation model) $C_{E+S+1}, \ldots, C_{E+S+O}$ by decision maker $D_j$.

![Figure 1. Membership function for importance weight of criteria (VL: (0, 0, 0.3), L: (0, 0.3, 0.5), M: (0.2, 0.5, 0.8), H: (0.5, 0.7, 1), VH: (0.7, 1, 1)).](image-url)
\(w_{ij}\) is defined as follows, where \(E\) denotes the number of economic criteria, \(S\) denotes the number of strategic justification criteria, \(O\) denotes the number of operational criteria and \(E + S + O = K\):

\[
\tilde{w}_{ij} = (m_{ij}, \alpha_{ij}, \beta_{ij}), \quad t = 1, 2, \ldots, k \\
j = 1, 2, \ldots, J.
\]

(3)

Linguistic weights assigned to criteria made to FMS alternatives by the decision-makers are aggregated by the mean operator. We define \(w_t\) as follows:

\[
\tilde{w}_t = \left(\frac{1}{J}\right) \otimes (\tilde{w}_{t1} \oplus \tilde{w}_{t2} \oplus \ldots \oplus \tilde{w}_{tJ}), \quad t = 1, 2, \ldots, k.
\]

(4)

Step 4: Define for all the alternatives \(a_i, a_{i'} \in A\) the fuzzy outranking-relation \(\tilde{\pi}\):

\[
\tilde{\pi}(a_i, a_{i'}) = \sum_{t=1}^{K} \tilde{w}_t \otimes p_t\left(\tilde{f}(a_i) - \tilde{f}(a_{i'})\right).
\]

(5)

With

\[f_k(a_i) = (m, \alpha, \beta) \quad \text{and} \quad f_k(a_{i'}) = (m, \gamma, \delta).\]

The degree of preference for the comparison of alternatives \(a_i\) and \(a_{i'}\) with regard to index \(f_k\) can be derived according to (for the extension of the preference from real numbers to fuzzy intervals, Figure 2):

\[
p_t\left(\tilde{f}_i(a_i) - \tilde{f}_i(a_{i'})\right) = p_t((m, \alpha, \beta) - (l, \gamma, \delta))
\]

\[= p_t(m - l, \alpha + \gamma, \beta + \delta)\]

\[= (p_t(m - l), (p_t(m - l) - p_t(m - l - \alpha - \gamma)))\]

\[= (m^{\tilde{p}_t}; \alpha^{\tilde{p}_t}; \beta^{\tilde{p}_t}).\]

(6)

Next, the degrees of preference are multiplied by the respective weights for each index:

\[
\tilde{w}_t \otimes p_t\left(\tilde{f}_i(a_i) - \tilde{f}_i(a_{i'})\right) = (m^{\tilde{w}_t}; \alpha^{\tilde{w}_t}; \beta^{\tilde{w}_t}) \otimes (m^{p_t}; \alpha^{p_t}; \beta^{p_t})
\]

\[\approx (m^{\tilde{w}_t \cdot m^{p_t}}; m^{\tilde{w}_t \cdot \alpha^{p_t}} + m^{p_t \cdot \alpha^{\tilde{w}_t}}; m^{\tilde{w}_t \cdot \beta^{p_t}} + m^{p_t \cdot \beta^{\tilde{w}_t}}).\]

(7)

Figure 2. Preference function applied to the difference of the triangular type fuzzy values \(f_i(a_i)\) and \(f_i(a_{i'}).\)
As the last step of defining the outranking relation $\tilde{\pi}$, the weighted preference degrees which have been calculated for each index $t$ are added:

$$\tilde{\pi}(a_i, a_j) = \sum_{t=1}^{k} \tilde{w}_t \otimes p_t \left( \tilde{f}_t(a_i) - \tilde{f}_t(a_j) \right)$$

$$\approx \sum_{t=1}^{k} \left( m^{w_t} \cdot m^{p_t} \cdot \alpha^{p_t} + m^{w_t} \cdot \alpha^{w_t} ; m^{w_t} \cdot \beta^{p_t} + m^{p_t} \cdot \beta^{w_t} \right)$$

$$= \left( \sum_{t=1}^{k} m^{w_t} \cdot m^{p_t} \cdot \alpha^{p_t} + m^{w_t} \cdot \alpha^{w_t} ; \sum_{t=1}^{k} m^{w_t} \cdot \beta^{p_t} + m^{p_t} \cdot \beta^{w_t} \right)$$

$$= (m^{\pi} ; \alpha^{\pi} ; \beta^{\pi}) .$$

\begin{equation}
\text{Step 5: As a measure for the strength of the alternatives } a_i \in A, \text{ the fuzzy leaving flow } \Phi^+(a_i) \text{ is calculated:}
\end{equation}

$$\Phi^+(a_i) = \frac{1}{T-1} \sum_{\ell=1}^{T} \tilde{\pi}(a_i, a_\ell), \quad \forall a_i \in A .$$

\begin{equation}
\text{Step 6: As a measure for the weakness of the alternatives } a_i \in A, \text{ the fuzzy entering flow } \Phi^-(a_i) \text{ is calculated:}
\end{equation}

$$\Phi^-(a_i) = \frac{1}{T-1} \sum_{\ell=1}^{T} \tilde{\pi}(a_\ell, a_i), \quad \forall a_i \in A .$$

\begin{equation}
\text{Step 7: Calculate fuzzy suitability index values for each FMS:}
\end{equation}

$$\tilde{\Phi}^{\text{net}}(a_i) = \Phi^+(a_i) - \Phi^-(a_i), \quad \forall a_i \in A .$$

3. Applied provident indices

A dynamic environment and its potential effect on alternatives can cause to change the value of some decision making indices during the time. For example, availability of all systems is near 100% at first but after a while it can be different for different systems. Especially when evaluation complexity, cost and risk increase, it is difficult to make a proper decision because a change in decision will be almost impossible. In fact it is necessary to consider future requirements and changes as well as current evaluation and it may be difficult when several indices are considered.

In this study the concept of provident indices is presented to solve this problem. Provident indices are future based and for calculating them at first the value of each provident index should be predicted for $P$ next periods based on historical data of alternatives or judgment of experts. Then for each period a special weight with regard to its importance should be dedicated. The importance can be determined by product features such as PLC, demand, etc. or competitiveness and potential competitors situation in the future. If $w_p$ and $c_{kp}$ indicate assigned weight and $k$th provident index value to the $p$th period respectively then:

$$C_k = w_1 c_{k1} + w_2 c_{k2} + \cdots + w_p c_{kp},$$

indicates the integrate value for the $k$th provident index.
4. Applied fuzzy indices

The process of assigning membership functions to fuzzy variables is either intuitive or based on some algorithmic or logical operations. Intuition is simply derived from the capacity of experts to develop membership functions through their own intelligence and judgment. Inference, rank ordering, angular fuzzy sets, neural networks, genetic algorithms, inductive reasoning, soft partitioning, and fuzzy statistics can be listed among the other methods stated in the literature to assign membership functions to fuzzy variables.

Ross (1995) argues that the precise shapes of the membership functions are not that important in their utility, while the approximate placement of the membership functions on the universe of discourse, the number of partitions used, and the overlapping character are of vital importance for application purposes in fuzzy operations.

The triangular membership functions are chosen for application considering their intuitive and computational-efficient representation and ease in computation. Nevertheless, the algorithm presented herein is independent of the type of membership functions, and thus, trapezoidal or parabolic membership functions that are determined using the membership identification techniques discussed above could also be used.

4.1 Operational indices

Some of these indices such as floor space are simply measurable but most of them can be investigated by simulation models. However, when the system complexity increases, it is difficult to optimise the system through an analytical approach; moreover, literature indicates the use of simulation as a more appropriate tool.

The variability of the manufacturing system dynamics generally depends on a large number of parameters, and on the impact that these have on the performance of the plant. A model developed to investigate these impacts cannot perfectly reproduce all the events of the real system, but it can consider a fair number of elements of variability, which should be identified and analysed statistically (D’Angelo et al. 1996).

The dynamics of an FMS and its stochastic characteristics require a specific approach for the definition of a final optimal configuration based on appropriate statistical analysis performed on data obtained from a simulation model. Such a model may provide solutions with a high degree of robustness and reliability, comparable with those obtained by direct experimentation, but with a low computational cost (D’Angelo et al. 1998).

For example, some of the outputs consisted of improvements in qualitative factors, work-in-process (WIP), percentage of tardy jobs, and yield (yield is defined as throughput minus scrap and rework). These output measures can be obtained by simulating different manufacturing systems.

By fuzzifying the output of simulation results a great amount of information from the simulation model can be retrieved for evaluation. The next steps are needed to fuzzify the outputs of the simulation model (Azadeh et al. 2007):

1. Define the operational indices.
2. Simulate the FMS alternatives being studied.
3. Execute simulation models for 2000 hours (250 working days, each day composed of three shifts, each shift eight hours of operation). Each model is also replicated 50 runs to ensure reasonable estimates of means of all outputs. Then the data related to operational indices would be extracted from simulation reports.
(4) Fuzzify the obtained data. For this, consider them as \((\mu - \sigma, \mu, \mu + \sigma)\) where \(\mu, \sigma\) are arithmetic mean and standard deviation of each index respectively.

4.2 Economic index

The economic aspects of the FMS selection process are addressed using the fuzzy discounted cash flow analysis. Fuzzy discounted cash flow analysis has been recently used by several authors as an alternative to the conventional cash flow models, where deterministic cash flows and discount rates or cash flow estimates and/or discount rates accompanied by probability distributions are used. Fuzzy set theory enables us to employ fuzzy cash flows that better account for the imprecision and vagueness in human judgments about the future, in place of cash flows defined as crisp numbers or probability distributions (Perego and Rangone 1998).

Since FMS alternatives are expected to provide an approximately equal production level, the annual operating revenues will not be considered while calculating the economic figure of merit. In this paper, the initial investment \((C_0)\), and annual operating and maintenance cost estimates of the FMS proposals for the future periods planning horizon are used.

Triangular fuzzy numbers appear as useful means of quantifying the uncertainty due to vagueness regarding cash flows, interest rates and inflation rates. The reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak and Tolga 2001). For instance, assuming that an expert’s estimate about the operating cost of a manufacturing investment is around two million dollars, it can be represented by a triangular fuzzy number as \((1.8, 2, 2.15)\).

A fuzzy provident index is applicable to economic justification of FMS investments in inflation-prone economies when incremental (when compared to the existing manufacturing system) cash flows in the economic evaluation of the FMS alternatives are considered.

To determine the provident fuzzy after-tax cost \((PFC)\) after inflation-adjusting cost items (considering specific rate of inflation) in each period \((\tilde{C}_p)\), the following equation can be used:

\[
PFC = \tilde{C}_0 \oplus [\tilde{C}_1 \otimes w_1] \oplus \cdots \oplus [\tilde{C}_p \otimes w_p].
\]

Where:

\[
\tilde{C}_p = \left( C_{pa}(1 - t) + \frac{tD_p}{\prod_{k=1}^{p}(1 + f_{ka})}, C_{pb}(1 - t) + \frac{tD_p}{\prod_{k=1}^{p}(1 + f_{kb})}, C_{pc}(1 - t) + \frac{tD_p}{\prod_{k=1}^{p}(1 + f_{kc})} \right),
\]

\(p = 1, 2, \ldots P;\)

and:

\(\tilde{C}_p:\) is fuzzy after tax cost in \(p\)th period;

\(w_j:\) is weight given to \(p\)th period regarding its importance (it can be considered as a fuzzy number if it is needed);

\(\tilde{C}_j = (C_{pa}, C_{pb}, C_{pc}):\) is end of \(p\)th period fuzzy predicate operating cost such as skilled labour cost and energy cost which is presented by a positive triangular fuzzy number;

\(f_j = (f_{pa}, f_{pb}, f_{pc}):\) is fuzzy general rate of inflation of \(p\)th period and it is a time dependent predicate;
4.3 Strategic indices

In order to determine the suitability of FMS alternatives versus the strategic indices, the linguistic values or provident indices can be used. The provident index concept was explained in Section 3. In this case the same as cost in the previous section, fuzzy numbers appear as useful means of quantifying the uncertainty due to vagueness regarding the strategic indices in the future periods.

A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterisation of phenomena that are too complex or ill-defined to be described in conventional quantitative terms (Zadeh 1975). Linguistic variables are introduced to represent the value of natural or artificial languages such as ‘around’, ‘very’, ‘little’, etc. The value of a linguistic variable can be quantified and extended to mathematical operations using the fuzzy set theory.

The linguistic values are defined by the variable set \( A = \{VP, P, F, G, VG\} \). Here, VP, P, F, G, VG denote ‘very poor’, ‘poor’, ‘fair’, ‘good’, and ‘very good’, respectively. The membership functions of the linguistic values are shown in Figure 3. These membership functions have previously been used in research articles pertaining to application of fuzzy sets for robot selection and measuring manufacturing competence (Liang and Wang 1993, Azzone and Rangone 1996).

Suppose that \( A_{ijt} \) is the linguistic rating assigned to FMS alternative FMS\( i \) by DM \( D_j \) for the strategic justification index \( C_t \). \( A_{ij} \) is defined below, where \( S \) denotes the number of strategic justification index. Linguistic rating assessments made to FMS alternatives by the DMs are aggregated by the mean operator.

\[
A_{ij} = (\alpha_{ij}, \beta_{ij}), \quad j = 1, 2, \ldots, J
\]
\[
i = 1, 2, \ldots, n
\]
\[
t = 2, 3, \ldots, S + 1
\]

\[
A_{it}(1/J) \otimes (A_{it1} \oplus A_{it2} \oplus \cdots \oplus A_{itJ}), \quad t = 2, 3, \ldots, S + 1.
\]

Here, \( A_{it} \) is the rating of FMS alternative FMS\( i \) versus the average linguistic ratings of FMS alternative FMS\( i \) for the strategic justification index \((C_2, \ldots, C_{1+S})\).

\[D_p: \text{depreciation expense in } p\text{th period;}
\]
\[P: \text{is planning horizon;}
\]
\[t: \text{is tax rate.}
\]
5. Proposed algorithm

In order to determine the optimal FMS the following procedure is devised.

Step 1: Construct a group of $J$ DMs ($D_1, D_2, \ldots, D_J$), who assess the importance weights of $K$ indices in period $P$ and suitability of $n$ prospective FMS alternatives under each of the $K$ indices including economic index ($PFC$), strategic justification indices $C_2, \ldots, C_{1+S}$, and operational indices (obtained from simulation model) $C_{S+2}, \ldots, C_{S+O+1}$ ($=K$).

Step 2: Determination of the indices to be considered (economic/strategic/operational).

Step 3: Calculate PFC (the economic index value by note of the determined incremental expected cash flows, interest rate and inflation rate by DMs).

Step 4: Identify a suitable value for each alternative relative to each strategic index.

Step 5: For operational indices, develop and run simulation models. Then collect and fuzzify the value relative to each operational index.

Step 6: Use fuzzy PROMETHEE to obtain fuzzy suitability index values of the FMS alternatives.

Step 7: Apply homogenous test as follows:

Step 7-1: Attributes which are applied in this method are the defuzzification of the fuzzy leaving and entering flows and fuzzy suitability index values for each FMS. For defuzzification, an approach based on the centre of area (COA) approach is selected:

$$x_{\text{defuzzy}} = \frac{\int x \mu(x) dx}{\int \mu(x) dx} = \frac{\int_{m-a}^{m} (1 - \frac{x-a}{a}) \cdot x dx + \int_{m}^{m+b} (1 - \frac{x-m}{b}) \cdot x dx}{\int_{m-a}^{m} (1 - \frac{x-a}{a}) dx + \int_{m}^{m+b} (1 - \frac{x-m}{b}) dx}$$

$$= \frac{\alpha m + \beta m + \frac{1}{2}(\beta^2 - \alpha^2)}{\alpha + \beta}.$$ (16)

The COA approach promises more reasonable results in the given decision situation than the mean of maxima (MOM) or maxima-method (MAX) and allows a consistent evaluation of trapezoidal and triangular fuzzy data as well as of crisp data. Furthermore, no additional parameters are needed (e.g., as for $\tilde{\delta}$-cuts).

Step 7-2: It is assumed there are three index values ($\Phi^-, \Phi^+$, and $\Phi^{\text{net}}$) and $n$ FMS alternatives which can be shown by a $(n \times 3)$ matrix named $Q$.

Step 7-3: $Q=[q_{ij}]_{n \times 3}$ is standardised such that all columns have a mean of 0 and variance of 1.

Step 7-4: The distance of every two FMS alternatives for each suitability index is computed. This is done to homogenise the decision making units (DMUs). Therefore, the distance matrix $D=[d_{ij}]_{n \times n}$ and vector
\( d = [d_i]_{i=1}^{n} \) where \( d_i \) is the minimum of \( i \)th row of matrix \( D \) are identified. To identify homogenous DMUs, the upper \((L_1)\) and lower \((L_2)\) limits of vector \( d \) is computed as \( L_1 = \bar{d} + 2sd \) and \( L_2 = \bar{d} - 2sd \) where \( \bar{d} \) and \( sd \) are the mean and standard deviation of vector \( d \) respectively. If all \( d_{ij} \) are within \( L_1 \) and \( L_2 \), homogeneity is achieved, follow the next step (Step 8) for ranking them otherwise, do Steps 9–12.

**Step 8:** Ranking the alternatives: because of the fact that DMUs are homogenous, clustering cannot be useful. Therefore a ranking procedure is required to determine the ranking order of the FMS alternatives with respect to their fuzzy suitability index values. There exist a number of papers concentrating on the ranking of fuzzy sets, and in particular fuzzy numbers. Bortolan and Degani (1985) present a comparative review of some of the methods for ranking fuzzy sets. They indicate that the methods generally provide consistent results for the simple examples while a number of problems are observed for the questionable cases. Jain (1976) utilised the concept of maximising set, which takes into account both the maximum utility associated with various alternatives and the grade of membership of the utilities. A more sensitive rule, which also considers measuring the left trend by utilising the minimising set concept, is applied in Chen’s (1985) method. In this work, Chen’s method for ranking fuzzy numbers is used, considering the consistency and ease of implementation.

Let \( A_i = (a_i, b_i, c_i) \) be a triangular fuzzy number with the membership function given as follows:

\[
f_{A_i}(x) = \begin{cases} 
0, & \text{for } x < a_i \text{ or } x > c_i \\
(x - a_i)/(b_i - a_i) & \text{for } a_i \leq x \leq b_i \\
(x - c_i)/(b_i - c_i) & \text{for } b_i \leq x \leq c_i 
\end{cases} \quad (17)
\]

The membership functions of maximising set \( M \) and minimising set \( G \) are given in Chen’s (1985) method as:

\[
f_{M}(x) = \begin{cases} 
\left[(x - x_{\min})/(x_{\max} - x_{\min})\right]^k, & \text{for } x_{\min} \leq x \leq x_{\max} \\
0, & \text{otherwise}
\end{cases} \quad \text{for } x_{\min} \leq x \leq x_{\max}
\]

\[
f_{G}(x) = \begin{cases} 
\left[(x - x_{\max})/(x_{\min} - x_{\max})\right]^k, & \text{for } x_{\min} \leq x \leq x_{\max} \\
0, & \text{otherwise}
\end{cases}
\]

Where:

\( S = \bigcup_{i=1}^{n} S_i \) (\( S_i \) is the support of \( A_i \));
\( x_{\max} = \sup(S) \) and \( x_{\min} = \inf(S) \).

The linear case is given by \( k = 1 \), while \( k > 1 \) represents risk-prone (convex) membership functions, and \( 0 < k < 1 \) represents risk-averse (concave) membership functions. Here, the value of \( k \) is assigned to be 1. When \( k = 1 \), the ranking value of \( A_i \) is calculated using the following expression:

\[
U_T(i) = \frac{1}{2} \left[ \frac{c_i - x_{\min}}{(x_{\max} - x_{\min}) - (b_i - c_i)} + 1 - \frac{x_{\max} - a_i}{(x_{\max} - x_{\min}) - (b_i - a_i)} \right], \quad i = 1, 2, \ldots, n \quad (19)
\]
Using Equation (15), the ranking of the \( n \) triangular fuzzy numbers can be obtained on the basis of their respective \( U_T(i) \). Sensitivity analysis of the chosen weights can be done for the first two FMSs after ranking them (Step 12). So the DM can select an FMS with more suitability and lower sensitivity with respect to chosen weights.

**Step 9:** Besides, it is important to note that a DMU with a simple radial efficiency score of one is not always the best one in overall uncertainties. For this reason, final ranking of DMUs are clustered by fuzzy C-Means method. For this, first the best number of clusters (\( x \)) is determined. Then, by considering defuzzification of the fuzzy leaving and entering flows and fuzzy suitability index values for each FMS, the fuzzy C-Means algorithm runs for the best number of clusters.

**Step 10:** Each of the clusters indicates a degree of desirability for the problem. To find the best cluster, calculate the weighted average of DMUs’ score for each of the clusters and each property. It should be noted that some units may belong to two or more clusters. To calculate the weighted average for each cluster, the degree of membership in each of the clusters is used as follows:

\[
\text{Avg}_j = \frac{\sum D_{ij} \cdot \Phi_{ij}^{\text{net}}}{\sum D_{ij}^{\text{net}}} , \quad 1 \leq j \leq x.
\]

where \( D_{ij} \) is the DMU_i’s degree of membership in the \( j \)th cluster and \( \Phi_{ij}^{\text{net}} \) is the DMU_i’s suitability index value (after defuzzification) in the \( j \)th cluster.

**Step 11:** The alternatives which belong to the cluster with high average values are appropriate for the problem. For FMSs which belong to this cluster sensitivity analysis of the chosen weights will be done to recognise an FMS with more suitability and lower sensitivity with respect to chosen weights.

**Step 12:** Sensitivity analysis of the chosen weights: good comprehensibility should be considered as an important advantage of this sensitivity analysis, since it can focus the discussion of the DMs on the most relevant, i.e., sensitive, criteria for the overall decision.

The graphical representation of the outranking graph and the sensitivity analysis of chosen weights are possible with less effort intervals and suitable means for the final decision which must be left to political and industrial DMs. For this, defuzzification (by COA) of ranking values \( (U_T(i)) \) for each FMS \( (i = 1, 2, \ldots, n) \) is necessary.

The sensitivity intervals give the range of values that the weight of one index can take without altering the results given in the initial set of weights, all other weights being kept constant. The narrower the interval boundaries, the more sensitive is the weighting of the respective index. This investigation is possible, since the PROMETHEE algorithm is basically an additive method.

Where the weight for the investigated index equals 100\%, the preference index can be defined as a uni-index preference index (with the only index as the investigated one). On the contrary, the preference index can also be calculated for all of the indices except for the investigated index.

Although it has been argued above that the net preference flow makes use of the concept of complete compensation and is therefore of less use in several
decision situations, it is to be stressed that the ranking values \( (U_T(i)) \) are a suitable mean for an intuitive sensitivity analysis: both the sensitivity interval and the resulting rank alterations can be shown in one graphical representation and can easily be explained to the DM.

6. An illustrative example

A practical case study illustrates the effectiveness of the proposed methodology.

**Step 1:** In this example a group of three decision-makers \( (D_1, D_2, D_3) \) is identified to evaluate the five FMS alternatives to replace the existing job shop.

**Step 2:** Floor space requirements (1000 ft\(^2\)), increase in process flexibility which indicates the ability to produce a given set of part types in several ways, improvement in quality factors and average percentage of availability & reliability (provident index) are considered as strategic benefits along with the incremental cost and benefits reduced to monetary terms.

**Step 3:** The initial investment, and annual operating and maintenance cost estimates of the five FMS proposals for the five-year planning horizon are given as triangular fuzzy numbers. The tax rate \( (t) \) is taken to be 15%. Consequently, expert estimates for the price level escalations related to the operating cost of the FMS alternatives, and general increase in the price levels for the five-year period are obtained.

The fuzzy after-tax cost \( (\tilde{C}_p) \) is calculated for the five FMS proposals in each period and the results with dedicated weight to each period are illustrated in Table 2. Also, calculated \( PFC \) index values for economic justification of FMS proposals are shown in Table 3.

**Step 4:** In this evaluation four strategic indices are considered. Fuzzy numbers are used by DMs for quantifying the uncertainty due to vagueness regarding the floor space requirements and improvement in quality factors. The average fuzzy evaluation of the FMS alternatives is performed under these strategic indices and shown in Table 4.

DMs use the linguistic variable set \( A = \{VP, P, F, G, VG\} \) to assess the suitability of the five FMS alternatives under the strategic index of increase in process flexibility. The average fuzzy evaluation of the FMS alternatives concerning this index is performed by using the linguistic assessments and is demonstrated in Table 5.

Average percentage of availability & reliability as a provident index is estimated for the next five years and the integrated value is calculated for the five FMS alternatives as explained in Section 3. The data and results are represented in Tables 6 and 7.

**Step 5:** As mentioned, the operational indices as statistical information outputs are collected from simulation models relative to FMS alternatives. These indices consist of improvements in WIP, improvement in percentage of tardy jobs and yield (yield is defined as throughput minus scrap and rework). The data is fuzzified for FMS proposals using the steps presented in Section 4.1 and shown in Table 8.

**Step 6:** The algorithm for fuzzy PROMETHEE described in Section 2 is applied to the evaluation tables, using the indicated preference function given in Figure 4 and the respective parameters \( (p \text{ and } q) \) for indices are presented in Table 9.
Table 2. The fuzzy after-tax cost ($\tilde{C}$) of the FMS proposals (in millions of dollars).

<table>
<thead>
<tr>
<th>Year ($p$)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>(105, 105, 105)</td>
<td>(12, 12.3, 12.6)</td>
<td>(12.1, 12.4, 12.8)</td>
<td>(12.5, 12.7, 13.4)</td>
<td>(13, 13.3, 13.6)</td>
<td>(13.5, 14, 14.4)</td>
</tr>
<tr>
<td>FMS2</td>
<td>(45, 45, 45)</td>
<td>(16, 16.5, 16.8)</td>
<td>(16.5, 17, 17.5)</td>
<td>(17, 17.6, 18)</td>
<td>(18, 18.6, 18.7)</td>
<td>(18, 18.7, 19)</td>
</tr>
<tr>
<td>FMS4</td>
<td>(35.4, 35.4, 35.4)</td>
<td>(17.6, 18, 18.3)</td>
<td>(18, 18.5, 18.8)</td>
<td>(19, 20, 20.5)</td>
<td>(21, 22, 22.8)</td>
<td>(23.1, 24, 25)</td>
</tr>
<tr>
<td>FMS5</td>
<td>(156.8, 156.8, 156.8)</td>
<td>(10, 10.3, 10.5)</td>
<td>(10.3, 10.5, 11)</td>
<td>(11, 11.2, 11.4)</td>
<td>(11.2, 11.7, 12)</td>
<td>(12, 12.4, 12.8)</td>
</tr>
<tr>
<td>Weight</td>
<td>–</td>
<td>0.176</td>
<td>0.235</td>
<td>0.294</td>
<td>0.176</td>
<td>0.118</td>
</tr>
</tbody>
</table>
Table 3. PFC index for economic justification of FMS proposals.

<table>
<thead>
<tr>
<th>FMS proposal</th>
<th>PFC ((t = 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>((117.523, 117.818, 118.271))</td>
</tr>
<tr>
<td>FMS2</td>
<td>((62.106, 62.468, 62.912))</td>
</tr>
<tr>
<td>FMS3</td>
<td>((111.676, 112.059, 112.535))</td>
</tr>
<tr>
<td>FMS4</td>
<td>((54.753, 55.518, 56.047))</td>
</tr>
<tr>
<td>FMS5</td>
<td>((167.612, 167.818, 168.165))</td>
</tr>
</tbody>
</table>

Table 4. The floor space requirements and improvement in quality factors of the FMS proposals.

<table>
<thead>
<tr>
<th>FMS proposal</th>
<th>Floor space requirements (1000 ft(^2)) ((t = 2))</th>
<th>Improvement in quality factors (%) ((t = 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>((5.3, 6, 6.5))</td>
<td>((26, 28, 30))</td>
</tr>
<tr>
<td>FMS2</td>
<td>((5, 5.4, 6))</td>
<td>((8, 10, 11))</td>
</tr>
<tr>
<td>FMS3</td>
<td>((4.9, 5.2, 5.4))</td>
<td>((19, 20, 22))</td>
</tr>
<tr>
<td>FMS4</td>
<td>((7.6, 8.8, 8.4))</td>
<td>((4, 5, 5.5))</td>
</tr>
<tr>
<td>FMS5</td>
<td>((7.7, 7.1, 7.5))</td>
<td>((40, 43, 50))</td>
</tr>
</tbody>
</table>

Table 5. The DMs evaluation of FMS proposals and the average linguistic ratings regarding increase in process flexibility.

<table>
<thead>
<tr>
<th>FMS proposal</th>
<th>DM1</th>
<th>DM2</th>
<th>DM3</th>
<th>Improvement in quality factors (%) ((t = 4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>F</td>
<td>F</td>
<td>G</td>
<td>((0.567, 0.767, 0.9))</td>
</tr>
<tr>
<td>FMS2</td>
<td>F</td>
<td>P</td>
<td>F</td>
<td>((0.2, 0.4, 0.6))</td>
</tr>
<tr>
<td>FMS3</td>
<td>F</td>
<td>G</td>
<td>VG</td>
<td>((0.5, 0.7, 0.9))</td>
</tr>
<tr>
<td>FMS4</td>
<td>F</td>
<td>G</td>
<td>G</td>
<td>((0.4, 0.6, 0.8))</td>
</tr>
<tr>
<td>FMS5</td>
<td>G</td>
<td>G</td>
<td>VG</td>
<td>((0.667, 0.867, 1))</td>
</tr>
</tbody>
</table>

Table 6. The fuzzy average percentage availability & reliability of the FMS proposals (%).

<table>
<thead>
<tr>
<th>Year ((p))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>((93, 95, 96))</td>
<td>((83, 85, 88))</td>
<td>((88, 91, 95))</td>
<td>((77, 80, 84))</td>
<td>((80, 85, 90))</td>
</tr>
<tr>
<td>FMS2</td>
<td>((90, 92, 94))</td>
<td>((73, 75, 80))</td>
<td>((80, 85, 88))</td>
<td>((63, 65, 68))</td>
<td>((70, 75, 80))</td>
</tr>
<tr>
<td>FMS3</td>
<td>((93, 95, 96))</td>
<td>((85, 90, 95))</td>
<td>((75, 80, 85))</td>
<td>((67, 70, 76))</td>
<td>((58, 60, 65))</td>
</tr>
<tr>
<td>FMS4</td>
<td>((90, 92, 94))</td>
<td>((80, 85, 88))</td>
<td>((73, 75, 80))</td>
<td>((58, 60, 65))</td>
<td>((50, 55, 60))</td>
</tr>
<tr>
<td>FMS5</td>
<td>((95, 98, 100))</td>
<td>((92, 95, 98))</td>
<td>((87, 90, 94))</td>
<td>((80, 85, 90))</td>
<td>((77, 80, 84))</td>
</tr>
<tr>
<td>Weight</td>
<td>0.176</td>
<td>0.235</td>
<td>0.294</td>
<td>0.176</td>
<td>0.118</td>
</tr>
</tbody>
</table>
Table 7. Provident fuzzy strategic (PFS) index of availability & reliability for FMS proposals.

<table>
<thead>
<tr>
<th>FMS proposal</th>
<th>PFS $(t = 5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>(84.823, 87.647, 91)</td>
</tr>
<tr>
<td>FMS2</td>
<td>(75.941, 79.176, 82.706)</td>
</tr>
<tr>
<td>FMS3</td>
<td>(77.118, 80.882, 85.353)</td>
</tr>
<tr>
<td>FMS4</td>
<td>(72.294, 75.353, 79.353)</td>
</tr>
<tr>
<td>FMS5</td>
<td>(87.176, 90.529, 94.118)</td>
</tr>
</tbody>
</table>

Table 8. Operational indices of the FMS proposals.

<table>
<thead>
<tr>
<th>FMS proposal</th>
<th>Improvements in WIP $(t = 6)$</th>
<th>Improvement in percentage of tardy jobs $(t = 7)$</th>
<th>Yield $(100)$ $(t = 8)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>(34.5, 35, 35.5)</td>
<td>(11.5, 12, 12.5)</td>
<td>(22, 22.4, 22.8)</td>
</tr>
<tr>
<td>FMS2</td>
<td>(19.9, 20.1, 20.3)</td>
<td>(4.7, 5, 5.3)</td>
<td>(16.2, 16.5, 16.8)</td>
</tr>
<tr>
<td>FMS3</td>
<td>(35.2, 35.5, 35.8)</td>
<td>(8.8, 9, 9.2)</td>
<td>(24.4, 24.7, 25)</td>
</tr>
<tr>
<td>FMS4</td>
<td>(16.8, 17.4, 18)</td>
<td>(0, 0.1, 0.3)</td>
<td>(18.18.1, 18.2)</td>
</tr>
<tr>
<td>FMS5</td>
<td>(45.2, 45.6, 46)</td>
<td>(13.4, 14, 14.6)</td>
<td>(30.5, 31.1, 31.7)</td>
</tr>
</tbody>
</table>

Figure 4. The indicated preference function (index with linear preference and indifference area).

Table 9. The respective parameters of preference functions with regard to each of the indices.

<table>
<thead>
<tr>
<th>Indices</th>
<th>$p$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1</td>
<td>50</td>
<td>5</td>
</tr>
<tr>
<td>t2</td>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>t3</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>t4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>t5</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>t6</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>t7</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>t8</td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>
The DMs utilise the linguistic set of weights $W = \{VL, L, M, H, VH\}$ defined in Section 2 to identify the importance of indices. The aggregate weights for each index are calculated by grouping the linguistic assessments of the three DMs:

$$W_1 = (0.567, 0.8, 1);$$
$$W_2 = (0.3, 0.567, 0.867);$$
$$W_3 = (0.4, 0.633, 0.933);$$
$$W_4 = (0.567, 0.8, 1);$$
$$W_5 = (0.567, 0.8, 1);$$
$$W_6 = (0.3, 0.567, 0.867);$$
$$W_7 = (0.4, 0.633, 0.933);$$
$$W_8 = (0.5, 0.7, 1).$$

If these weights are taken into account, fuzzy PROMETHEE reveals the results given in Table 10. The higher the leaving flow and the lower the entering flow, the better the action. Figure 5 displays a suitable illustration of the outcome of fuzzy PROMETHEE for presentation to the DMs: since the entering flows $\Phi^-$ for the respective techniques are depicted with a negative sign, their indication of the relative weakness of the considered techniques becomes more evident. To give the DMs a quick overview, also the fuzzy net flows $\Phi^{\text{net}}$ are depicted in Figure 6 and offered as additional information. The partial pre-order is derived from both the ranking according to the leaving flows $\Phi^+$ (FMS5 > FMS1 > FMS3 > FMS2 > FMS4) and according to the entering flows $\Phi^-$ (FMS4 > FMS2 > FMS3 > FMS5 > FMS1).

Figure 5 is interpreted by the DMs. They may judge that FMS5, FMS1 and FMS3 are preferred since for these systems the leaving flows outweigh the entering flows significantly, while system FMS2 and FMS4 are in the low range. This result would seem to be reasonable. It could be argued that those systems with leaving flows almost equal or larger than the entering flows should be regarded as the ‘best considered systems’.

**Step 7:** Applying homogenous test.

**Step 7-1:**

$$Q = \begin{bmatrix}
FMS1 & 5.67917 & 7.043464 & 3.114752 \\
FMS2 & -0.07658 & 3.445038 & 9.399924 \\
FMS3 & 5.107852 & 6.239749 & 4.920448 \\
FMS4 & -1.53918 & 2.200988 & 9.547896 \\
FMS5 & 6.659243 & 8.823189 & 3.332054 
\end{bmatrix}$$

Step 7-2:

$$Q^{\text{standard}} = \begin{bmatrix}
0.678078 & 0.553842 & -0.92387 \\
-0.87494 & -0.78105 & 1.04567 \\
0.523925 & 0.255692 & -0.35804 \\
-1.26958 & -1.24254 & 1.092026 \\
0.942522 & 1.214056 & -0.85578 
\end{bmatrix}.$$
Table 10. Preference indices for FMS alternatives.

<table>
<thead>
<tr>
<th>FMS proposal</th>
<th>FMS1</th>
<th>FMS2</th>
<th>FMS3</th>
<th>FMS4</th>
<th>FMS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMS1</td>
<td>(0, 0, 0)</td>
<td>(1.987, 4.065, 6.542)</td>
<td>(0.440, 0.725, 2.358)</td>
<td>(1.824, 4.427, 6.562)</td>
<td>(0.657, 1.854, 2.004)</td>
</tr>
<tr>
<td>FMS2</td>
<td>(0.582, 1.722, 1.610)</td>
<td>(0, 0, 0)</td>
<td>(0.562, 1.362, 1.570)</td>
<td>(0.756, 1.719, 3.132)</td>
<td>(0.747, 2.029, 2.117)</td>
</tr>
<tr>
<td>FMS3</td>
<td>(0.0733, 0.467, 0.717)</td>
<td>(1.854, 3.654, 6.090)</td>
<td>(0, 0, 0)</td>
<td>(1.699, 4.467, 6.635)</td>
<td>(0.777, 2.087, 2.481)</td>
</tr>
<tr>
<td>FMS4</td>
<td>(0.567, 1.367, 1.754)</td>
<td>(0.308, 0.363, 1.122)</td>
<td>(0.567, 1.367, 2.134)</td>
<td>(0, 0, 0)</td>
<td>(0.567, 1.367, 1.567)</td>
</tr>
<tr>
<td>FMS5</td>
<td>(1.017, 2.485, 4.231)</td>
<td>(2.311, 4.844, 7.460)</td>
<td>(1.457, 3.067, 5.413)</td>
<td>(2.364, 5.273, 7.718)</td>
<td>(0, 0, 0)</td>
</tr>
</tbody>
</table>
Step 7-3:

\[
D = \begin{bmatrix}
0 & 2.841274 & 0.657895 & 3.329298 & 0.714458 \\
2.841274 & 0 & 2.236518 & 0.608992 & 3.301373 \\
0.657895 & 2.236518 & 0 & 2.750283 & 1.158202 \\
3.329298 & 0.608992 & 2.750283 & 0 & 3.836955 \\
0.714458 & 3.301373 & 1.158202 & 3.836955 & 0
\end{bmatrix}
\]

Step 7-4:

\[
d = 0.657895 \\
0.608992 \\
\]

\[L_1 = 0.737066 \quad \text{and} \quad L_2 = 0.562226.\]

Figure 5. Defuzzified preference flows and partial pre-order of FMS proposals.

Figure 6. Fuzzy preference flows of FMS proposals.
It can be seen that all components of vector $d$ are in the interval between $L_1$ and $L_2$, so homogeneity is achieved and we should follow the next step (Step 8). But for covering both branches of the algorithm, all next steps will be done.

**Step 8:** Ranking order of $\Phi^{\text{net}}$ fuzzy numbers.

Table 11 indicated the results of calculating the respective $U_T(i)$ for each FMS proposal.

To evaluate the effect of the provident indices which have been introduced in this paper two different sets of data, have been made. In the first set of data provident indices are used while the second set of data uses only the data of the first period from the provident indices for the evaluation calculation.

Table 12 shows the results of calculating respective $U_T(i)$ for each FMS proposal in this second set of data. It is apparent that when the algorithm is executed with the provident indices, the results are different. Thus, if provident indices were not used, some part of information would be lost and the final decision might be different.

**Step 9:** Clustering FMS proposals by fuzzy C-Means method.

First the best number of clusters should be determined. The values of partition coefficient (PC) and entropy value (PE) are shown in Figure 7. The ‘best’ number of clusters is the point on the horizontal axis ($c^*$) that the PE of $c^*$ lies below the rising trend and the value for the PC of $c^*$ lies above the falling trend. Figure 7 shows that, according to these two criteria, the best partitioning of the data is achieved with three clusters.

Then the fuzzy C-Means algorithm runs for three clusters (A, B and C). The degrees of membership of the FMS proposals in each of the three clusters (A, B and C) are shown in Figure 8. In Table 13 the FMS proposals which belong to each of these clusters are shown.
Step 10: Calculate weighted average of FMSs’ score for each of the clusters.

Each of the clusters indicates a degree of desirability for FMS proposals. To find the best cluster, calculate weighted average of FMSs’ rank for each of the clusters and features.

It should be noted that some units may belong to two or more clusters. For instance by note of the high degree of membership of FMS1 in clusters (A and B), this DMU is considered in both of these clusters. To calculate weighted average for each cluster, the
Figure 9. Sensitivity analysis of the chosen weights with regard to indices.
FMS proposals degree of membership in each of the clusters is used. For example, weighted average of cluster A in $\Phi^{\text{net}}$ is equal to:

\[
\frac{(\text{FMS5} + (\text{FMS1} \ast \text{FMS1 degree of membership in cluster A}))}{1 + \text{FMS1 degree of membership in cluster A}} = \frac{(6.659243 + (5.67917 \ast 0.65))}{1 + 0.65} = 6.274715.
\]

**Step 11:** Find the cluster with the best results.

The FMS proposals which belong to the cluster with high value averages in $\Phi^{\text{net}}$ and $\Phi^+$ and low value in $\Phi^-$ are appropriate. As can be seen in Table 13 cluster A includes the best FMS proposals.

**Step 12:** Sensitivity analysis of the chosen weights.

The most important information of the fuzzy PROMETHEE result is the partial pre-order, which also reveals incomparability and, even more, the sensitivity analysis which shows the most sensitive weights and index. The diagrams obtained by the sensitivity analysis (Figure 9) communicate how the ranking of alternatives responds to changes in the weights of the indices, the narrower a sensitivity interval, the more sensitive to the respective index and its weighting. In this case study, the $PFC$ as a provident index ($t = 1$) is more sensitive than the others. Also, the slope of the alternative gives an indication: here, system FMS5 quite often shows different results than the others.

For more illustration, one of the charts in Figure 9 is considered. Figure 9; chart A sketches the sensitivity analysis for an index 'provident fuzzy cost ($t = 1$)'. The original chosen weight of which resulting complete order of the investigated alternatives is FMS1 → FMS3 → FMS5 → FMS2 → FMS4. Beyond the upper boundary of the sensitivity interval, the rank order changes to FMS4 → FMS2 → FMS3 → FMS1 → FMS5, and beyond the lower boundary of the sensitivity interval the rank order alters to FMS5 → FMS1 → FMS3 → FMS2 → FMS4. The DM can easily grasp that for this case, alternative FMS5 is largely dependent on the weighting of the investigated index.

DMs have to keep in mind the assumptions underlying the data preparation. Firstly, the limited data availability due to the varying measurements restricts the reliability of the obtained results. Secondly, the subjective character of the setting of weights and of the preference parameters should be considered. With this final interpretation by the DMs it becomes evident that soft decision analysis only supports the decision, which itself is left to the DMs.

It is observed that the ranking order is FMS1 → FMS3 → FMS5 → FMS2 → FMS4. FMS1 appears to be the most suitable FMS proposal as a result of the proposed algorithm, and thus, is the first one to be considered for purchase. But FMS1 and FMS3 have a short distance and sensitivity analysis shows that by changing the weights of indices the result can be changed. So final selection, between these two systems should be left to the DMs.

**7. Conclusion**

Global competition in the manufacturing environment has forced firms to increase the quality and responsiveness to customisation, while lowering the costs. An FMS, when
properly implemented, provides major strategic and operating benefits to the manufacturing firm such as flexibility, improved product quality, and reduced lead time and WIP. In order to incorporate these notable benefits that cannot be reduced to monetary terms into the justification and selection process of the FMS, an integrated approach considering all economic, strategic and operational indices is required.

As an alternative to the traditional economic and strategic justification techniques in this paper a methodology based on fuzzy C-Means and fuzzy PROMETHEE methods, which bringing together flexibility and simplicity for the user, was proposed for the FMS selection problem by taking into account not only the economic index, but also the key strategic justification indices and operational indices obtained from a simulation model. In this study future oriented indices were presented which enable us to consider the effect of future changes in the index value during the decision making process. These future oriented indices were named provident indices. Thus, both a dynamic and a fuzzy environment were considered by using provident and fuzzy indices.

The fuzzy decision algorithm proposed here helps to resolve the vagueness in the FMS evaluation process by quantifying the non-monetary impacts. Our procedures identify an effective set of alternative FMSs from which the DM can choose. Provision of these alternative choices provides flexibility in the decision making process.

What is new in this paper is the incorporation of variability in provident and fuzzy measures in the performance of alternative FMSs, and in the application of the fuzzy C-Means method and fuzzy PROMETHEE methods and sensitivity analysis that provide the DM with effective alternative choices by identifying significant differences among FMS alternatives and appropriate choices through considered future periods, and presents graphic aids for better interpretation of results.

The most important information of the fuzzy PROMETHEE result is the partial pre-order, which also reveals incomparability and, even more, the sensitivity analysis which shows the most sensitive weights and indices. The good comprehensibility should be considered as an important advantage of this sensitivity analysis, since it could focus the discussion of the DMs on the most relevant, i.e., sensitive, index for the overall decision.

The methodology was illustrated through its application on a set of five FMS proposals and eight indices. The results showed that the provident indices had a distinguished effect on the results. Thus, if provident indices were not used, some part of information would be lost and the final decision might be different. Thus, results indicate that the proposed algorithm estimates more robust results.

References


