vertical plane and the horn return loss at the input port has been compared with measurements. For the second case, the results provided by the hybrid technique have been compared with those yielded by the CST Microwave Studio. In both cases, the radiation pattern and the return loss predicted by hybrid technique shows good agreement with the measurement and with the results provided by the CST Microwave Studio.

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Electromagnetic Characterization of Biaxial Bianisotropic Media Using the State Space Approach

Davoud Zarifi, Mohammad Soleimani, and Ali Abdolali

***Abstract*—The electromagnetic characterization method based on the state space approach, which has been proposed for parameter retrieval of uniaxial chiral composites is extended in order to extract the electromagnetic parameters of biaxial bianisotropic media as well. The properties of state transition matrix of a biaxial bianisotropic layer are presented as two theorems, and the formulation of the proposed electromagnetic characterization method is then provided. The proposed approach reduces nonlinearity and complexity of the problem by considering properties of the state transition matrix. The validity of the method is verified using analysis of a bianisotropic specimen. The results show that the proposed technique allows for a characterization at oblique incidences.**

***Index Terms*—Biaxial bianisotropic media, electromagnetic characterization, state space approach.**

I. Introduction

The interaction of electromagnetic waves with complex electromagnetic structures has generated an enormous research interest over the years. Due to the need for special electromagnetic properties, artificial structures, such as bi-isotropic and bianisotropic, classified as materials exhibiting magneto-electric coupling, are suggested. Bianisotropic media are studied for various applications such as waveguides [1], polarization transformers [2], absorbers [3] and backward wave media [4].

With the increased interest in artificial electromagnetic structures, various methods have been proposed for retrieving the effective electromagnetic parameters of such artificial structures [5]–[7]. A commonly used scattering parameter method is generally based on the inversion of reflection and transmission parameters of a plane wave incident on the structure to give electromagnetic parameters. Most attempts for electromagnetic characterization of materials at oblique incidence or accounting for anisotropy or bianisotropy have relied on simplifying assumptions or using fully numerical optimization methods and curve fitting techniques [8]–[11]. Therefore, it seems that for media where the material tensor parameters are complex, a new approach is needed.

The state space approach or 4×4 transition matrix method is a commonly used method for the analysis of scattering from planar layered generalized anisotropic or bianisotropic media [12]–[14], which its application in the inverse scattering problems has not been reported in the literature. Recently, a parameter retrieval technique based on the state space approach for the electromagnetic characterization of uniaxial chiral structures has been introduced [15]. The main difference between this approach and conventional retrieval methods is direct calculation of the propagation constant and impedance of the modes supported by the medium. This feature effectively allows avoiding nonlinearity and complexity of the problem.

The objective of this work is to characterize biaxial bianisotropic media using the state space approach. In [15], only normal incidence

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is considered, which prevents one from taking into account spatial dispersion. In the present work, some of constitutive parameters of bianisotropic layer are only active in oblique illuminations, and so for the characterization of the slab, scattering parameters corresponding to an oblique illumination are also required.

II. Forward Problem Analysis

In this section, we review the analysis of the problem of plane wave scattering from a biaxial bianisotropic layer which has tensorial forms of constitutive relations as [16]

$$\bar{D} = \bar{\bar{\varepsilon}}\bar{E} - j\sqrt{\varepsilon_0\mu_0}\,\bar{\bar{\kappa}}\bar{H} \tag{1}$$

$$\bar{B} = \bar{\bar{\mu}}\,\bar{H} + j\sqrt{\varepsilon_0\mu_0}\,\bar{\bar{\kappa}}\bar{E} \tag{2}$$

where ε_0 and μ_0 are permittivity and permeability of free space, respectively. In (1) and (2), $\bar{\bar{\varepsilon}} = \varepsilon_0\mathrm{diag}(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}), \bar{\bar{\mu}} = \mu_0\mathrm{diag}(\mu_{xx}, \mu_{yy}, \mu_{zz})$ and $\bar{\bar{\kappa}} = \mathrm{diag}(\kappa_{xx}, \kappa_{yy}, \kappa_{zz})$ are diagonal matrices. It is assumed that the TE or TM polarized plane waves are obliquely incident at angle θ_0 from free space to the slab, as shown in Fig. 1. According to phase matching, we write $\partial/\partial z = -jk_0\sin\theta_0$ where k_0 is the vacuum wave number. By substituting (1) and (2) into curl Maxwell's equations and by eliminating x-components of electric and magnetic fields, the equations describing the biaxial bianisotropic layer are given by

$$\frac{d}{dx}\begin{pmatrix}\bar{E}_T\\ \bar{H}_T\end{pmatrix} = \bar{\bar{\Gamma}}_\omega\begin{pmatrix}\bar{E}_T\\ \bar{H}_T\end{pmatrix} \tag{3}$$

where $\bar{E}_T = (E_z, E_y)$ and $\bar{H}_T = (H_z, H_y)$ are the transverse components of fields, respectively, and $\bar{\bar{\Gamma}}_\omega$ is given by

$$\bar{\bar{\Gamma}}_\omega = \frac{\omega}{c}\bar{\bar{\Gamma}} = \frac{\omega}{c}\begin{pmatrix} 0 & -(\kappa_{yy} + \kappa_{xx}(\sin(\theta_0))^2/Q) & 0 & j\eta_0(\mu_{yy} - \mu_{xx}(\sin(\theta_0))^2/Q)\\ \kappa_{zz} & 0 & -j\,\eta_0\mu_{zz} & 0\\ 0 & j\eta_0^{-1}(-\varepsilon_{yy} + \varepsilon_{xx}(\sin(\theta_0))^2/Q) & 0 & -(\kappa_{yy} + \kappa_{xx}(\sin(\theta_0))^2/Q)\\ j\,\eta_0^{-1}\varepsilon_{zz} & 0 & \kappa_{zz} & 0 \end{pmatrix} \tag{4}$$

where ω, c and η_0 are the angular frequency, the speed of light in free space, and the free space impedance, respectively, and $Q = \varepsilon_{xx}\mu_{xx} - \kappa_{xx}^2$. The state equations of more complex bianisotropic media under more general conditions are discussed in [13]. Defining 4×4 state transition matrix $\bar{\bar{\Phi}}$ as

$$\begin{aligned}\begin{pmatrix}\bar{E}_T(0)\\ \bar{H}_T(0)\end{pmatrix} &= \bar{\bar{\Phi}}\begin{pmatrix}\bar{E}_T(d)\\ \bar{H}_T(d)\end{pmatrix}\\ &= \begin{pmatrix}(\bar{\bar{\Phi}}_1)_{2\times2} & (\bar{\bar{\Phi}}_2)_{2\times2}\\ (\bar{\bar{\Phi}}_3)_{2\times2} & (\bar{\bar{\Phi}}_4)_{2\times2}\end{pmatrix}\begin{pmatrix}\bar{E}_T(d)\\ \bar{H}_T(d)\end{pmatrix}\end{aligned} \tag{5}$$

where $\bar{\bar{\Phi}} = \exp(-\bar{\bar{\Gamma}}_\omega d)$. By considering $\bar{E}_T^i(0) = \bar{\bar{\mathrm{Z}}}_0\bar{H}_T^i(0), \bar{E}_T^r(0) = -\bar{\bar{\mathrm{Z}}}_0\bar{H}_T^r(0), \bar{E}_T^t(d) = \bar{\bar{\mathrm{Z}}}_0\bar{H}_T^t(d)$ and after some simple manipulations on (5), one can write:

$$\bar{E}_T^i(0) + \bar{E}_T^r(0) = \bar{\bar{\Phi}}_1\,\bar{E}_T^t(d) + \bar{\bar{\Phi}}_2\,\mathrm{Z}_0^{-1}\,\bar{E}_T^t(d) \tag{6}$$

$$\mathrm{Z}_0^{-1}\left(\bar{E}_T^i(0) - \bar{E}_T^r(0)\right) = \bar{\bar{\Phi}}_3\,\bar{E}_T^t(d) + \bar{\bar{\Phi}}_4\,\mathrm{Z}_0^{-1}\,\bar{E}_T^t(d). \tag{7}$$

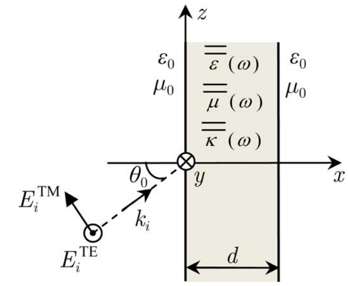

Fig. 1. A bianisotropic slab exposed to a linearly polarized plane wave.

where the superscripts i, r, and t denote the incident, reflected, and transmitted fields, respectively, and the wave impedance matrix $\bar{\bar{\mathrm{Z}}}_0$ is defined as

$$\bar{\bar{\mathrm{Z}}}_0 = \begin{pmatrix}0 & \eta_0\cos\theta_0\\ -\eta_0/\cos\theta_0 & 0\end{pmatrix}. \tag{8}$$

The reflection and transmission coefficients can be obtained after some simple matrix manipulations [12], [15]:

$$\begin{aligned}\begin{pmatrix}R_{zz} & R_{zy}\\ R_{yz} & R_{yy}\end{pmatrix} &= [\bar{\bar{\Phi}}_1\bar{\bar{\mathrm{Z}}}_0 + \bar{\bar{\Phi}}_2 - \bar{\bar{\mathrm{Z}}}_0(\bar{\bar{\Phi}}_3\bar{\bar{\mathrm{Z}}}_0 + \bar{\bar{\Phi}}_4)]\\ &\quad\times[\bar{\bar{\Phi}}_1\bar{\bar{\mathrm{Z}}}_0 + \bar{\bar{\Phi}}_2 + \bar{\bar{\mathrm{Z}}}_0(\bar{\bar{\Phi}}_3\bar{\bar{\mathrm{Z}}}_0 + \bar{\bar{\Phi}}_4)]^{-1}\end{aligned} \tag{9a}$$

$$\begin{pmatrix}T_{zz} & T_{zy}\\ T_{yz} & T_{yy}\end{pmatrix} = 2\,\bar{\bar{\mathrm{Z}}}_0[\bar{\bar{\Phi}}_1\bar{\bar{\mathrm{Z}}}_0 + \bar{\bar{\Phi}}_2 + \bar{\bar{\mathrm{Z}}}_0(\bar{\bar{\Phi}}_3\bar{\bar{\mathrm{Z}}}_0 + \bar{\bar{\Phi}}_4)]^{-1}. \tag{9b}$$

III. Properties of State Transition Matrix of a Biaxial Bianisotropic Slab

In this section, we introduce and prove two properties of state transition matrix of a biaxial bianisotropic layer which will be used for the proposed parameter retrieval procedure.

A. *Theorem 1*

The absolute values of the entries of state transition matrix of a biaxial bianisotropic slab and its inverse are equal. The state transition matrix of a biaxial bianisotropic slab and its inverse could be written as

$$\bar{\bar{\Phi}} = \begin{pmatrix}\boxed{\Phi_{11}} & \boxed{\Phi_{12}} & \boxed{\Phi_{13}} & \boxed{\Phi_{14}}\\ \boxed{\Phi_{21}} & \boxed{\Phi_{22}} & \boxed{\Phi_{23}} & \Phi_{13}\\ \boxed{\Phi_{31}} & \boxed{\Phi_{32}} & \Phi_{22} & \Phi_{12}\\ \boxed{\Phi_{41}} & \Phi_{31} & \Phi_{21} & \Phi_{11}\end{pmatrix}. \tag{10a}$$

$$\bar{\bar{\Phi}}^{-1} = \begin{pmatrix}\Phi_{11} & -\Phi_{12} & \Phi_{13} & -\Phi_{14}\\ -\Phi_{21} & \Phi_{22} & -\Phi_{23} & \Phi_{13}\\ \Phi_{31} & -\Phi_{32} & \Phi_{22} & -\Phi_{12}\\ -\Phi_{41} & \Phi_{31} & -\Phi_{21} & \Phi_{11}\end{pmatrix}. \tag{10b}$$

Observe that $\bar{\bar{\Phi}}$ has only 10 distinct elements. Its Proof is presented in Appendix.

B. *Theorem 2*

The determinant of state transition matrix of a biaxial bianisotropic slab is equal to unity. As proved in [15]:

$$\det(\bar{\bar{\Phi}}) = \exp\left[\sum_{\alpha=1}^{4}\Gamma_\omega(\alpha,\alpha)\right]. \tag{11}$$

According to (4), the sum of the diagonal elements of $\bar{\bar{\Gamma}}_\omega$ matrix of biaxial bianisotropic slab is zero, and so the determinant of its $\bar{\bar{\Phi}}$ is equal to unity.

IV. Theory and Formulation of Proposed Method

This section deals with the formulation of electromagnetic characterization procedure for the biaxial bianisotropic slabs.

It is clear that six tensor parameters $\varepsilon_{yy}, \varepsilon_{zz}, \mu_{yy}, \mu_{zz}, \kappa_{yy}$, and κ_{zz} are active when the slab is normally illuminated by TE or TM plane waves, while three other parameters $\varepsilon_{xx}, \mu_{xx}$ and κ_{xx} are active when the slab is obliquely illuminated. So, for retrieving all the constitutive tensor parameters of a biaxial bianisotropic slab, scattering parameters corresponding to the illuminations at two different angles of incidence are required.

A. Determination of State Transition Matrix

First, perpendicular polarization incidence (TE) can be considered in which the incident wave is a y-polarized wave with amplitude of 1 V/m. In this case, (6) and (7) may be rewritten as

$$\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}E^r_{z-TE}\\E^r_{y-TE}\end{pmatrix}=\left(\begin{pmatrix}\Phi^{\rm obl}_{11} & \Phi^{\rm obl}_{12}\\ \Phi^{\rm obl}_{21} & \Phi^{\rm obl}_{22}\end{pmatrix}\right.$$
$$\left.+\frac{1}{\eta_0}\begin{pmatrix}-\Phi^{\rm obl}_{14}/\cos\theta_0 & \Phi^{\rm obl}_{13}\cos\theta_0\\ -\Phi^{\rm obl}_{13}/\cos\theta_0 & \Phi^{\rm obl}_{23}\cos\theta_0\end{pmatrix}\right)\begin{pmatrix}E^t_{z-TE}\\E^t_{y-TE}\end{pmatrix} \tag{12}$$

$$\begin{pmatrix}\cos\theta_0\\0\end{pmatrix}-\begin{pmatrix}\cos\theta_0 E^r_{y-TE}\\ -E^r_{z-TE}/\cos\theta_0\end{pmatrix}=\left(\eta_0\begin{pmatrix}\Phi^{\rm obl}_{31} & \Phi^{\rm obl}_{32}\\ \Phi^{\rm obl}_{41} & \Phi^{\rm obl}_{31}\end{pmatrix}\right.$$
$$\left.+\begin{pmatrix}-\Phi^{\rm obl}_{12}/\cos\theta_0 & \Phi^{\rm obl}_{22}\cos\theta_0\\ -\Phi^{\rm obl}_{11}/\cos\theta_0 & \Phi_{21}\cos\theta_0\end{pmatrix}\right)\begin{pmatrix}E^t_{z-TE}\\E^t_{y-TE}\end{pmatrix} \tag{13}$$

where TE subscript in the reflected and transmitted electric fields denotes polarization of incident wave and obl superscript denotes oblique incidence. Similarly, consider the TM incident wave, for which (6) and (7) can be rewritten as

$$\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}E^r_{z-TM}\\E^r_{y-TM}\end{pmatrix}=\left(\begin{pmatrix}\Phi^{\rm obl}_{11} & \Phi^{\rm obl}_{12}\\ \Phi^{\rm obl}_{21} & \Phi^{\rm obl}_{22}\end{pmatrix}\right.$$
$$\left.+\frac{1}{\eta_0}\begin{pmatrix}-\Phi^{\rm obl}_{14}/\cos\theta_0 & \Phi^{\rm obl}_{13}\cos\theta_0\\ -\Phi^{\rm obl}_{13}/\cos\theta_0 & \Phi^{\rm obl}_{23}\cos\theta_0\end{pmatrix}\right)\begin{pmatrix}E^t_{z-TM}\\E^t_{y-TM}\end{pmatrix} \tag{14}$$

$$\begin{pmatrix}0\\-1/\cos\theta_0\end{pmatrix}+\begin{pmatrix}-\cos\theta_0 E^r_{y-TM}\\ E^r_{z-TM}/\cos\theta_0\end{pmatrix}=\left(\eta_0\begin{pmatrix}\Phi^{\rm obl}_{31} & \Phi^{\rm obl}_{32}\\ \Phi^{\rm obl}_{41} & \Phi^{\rm obl}_{31}\end{pmatrix}\right.$$
$$\left.+\begin{pmatrix}-\Phi^{\rm obl}_{12}/\cos\theta_0 & \Phi^{\rm obl}_{22}\cos\theta_0\\ -\Phi^{\rm obl}_{11}/\cos\theta_0 & \Phi^{\rm obl}_{21}\cos\theta_0\end{pmatrix}\right)\begin{pmatrix}E^t_{z-TM}\\E^t_{y-TM}\end{pmatrix}. \tag{15}$$

Notice that a set of eight linear equations is derived, while the number of distinct elements of the state transition matrix $\bar{\bar{\Phi}}^{\rm obl}$ is ten. By considering two discussed theorems in the previous section, the necessary equations are provided to uniquely determine the unknown entries of $\bar{\bar{\Phi}}^{\rm obl}$. Similarly, in the case of normal incidence, we can consider (12)–(15) assuming $\theta_0 = 0^\circ$ and the state transition matrix $\bar{\bar{\Phi}}^{\rm nor}$, where *nor* superscript denotes the normal incidence, can be fully determined.

B. Determination of Constitutive Tensor Parameters

Once the state transition matrices $\bar{\bar{\Phi}}(\bar{\bar{\Phi}}^{\rm nor}$ and $\bar{\bar{\Phi}}^{\rm obl})$ are determined, according to definition of $\bar{\bar{\Phi}} = \exp(-\bar{\bar{\Gamma}}_\omega d)$, $\bar{\bar{\Gamma}}$-matrices ($\bar{\bar{\Gamma}}^{\rm nor}$ and $\bar{\bar{\Gamma}}^{\rm obl}$) can be subsequently identified as

$$\bar{\bar{\Gamma}} = -\frac{c}{\omega d}\ln(\bar{\bar{\Phi}}) = -\frac{\lambda_0}{2\pi d}\ln(\bar{\bar{\Phi}}) \tag{16}$$

where λ_0 is the wavelength in free space. If the thickness of slab be much smaller than the free space wavelength, the Γ-matrix is unambiguously identified [15]. The constitutive parameters $\varepsilon_{yy}, \varepsilon_{zz}, \mu_{yy}, \mu_{zz}, \kappa_{yy}$, and κ_{zz} are active in the case of normal incidence, and so once the $\Gamma^{\rm nor}$-matrix is determined, we can simply write:

$$\begin{cases}\varepsilon_{yy} = j\eta_0\bar{\bar{\Gamma}}^{\rm nor}_{32} & \varepsilon_{zz} = -j\eta_0\bar{\bar{\Gamma}}^{\rm nor}_{41}\\ \mu_{yy} = -j\eta_0^{-1}\bar{\bar{\Gamma}}^{\rm nor}_{14} & \mu_{zz} = j\eta_0^{-1}\bar{\bar{\Gamma}}^{\rm nor}_{23}\\ \kappa_{yy} = -\bar{\bar{\Gamma}}^{\rm nor}_{12} & \kappa_{zz} = \bar{\bar{\Gamma}}^{\rm nor}_{21}\end{cases}. \tag{17}$$

Determination of other constitutive parameters, i.e., $\varepsilon_{xx}, \mu_{xx}$ and κ_{xx} is more complex, because the elements of $\bar{\bar{\Gamma}}^{\rm obl}$ are also required. Considering (4) for normal and oblique incidences, one can write:

$$\frac{\bar{\bar{\Gamma}}^{\rm obl}_{32}-\bar{\bar{\Gamma}}^{\rm nor}_{32}}{j\eta_0^{-1}\sin^2\theta_0} = \frac{\varepsilon_{xx}}{\varepsilon_{xx}\mu_{xx}-\kappa^2_{xx}} \tag{18}$$

$$-\frac{\bar{\bar{\Gamma}}^{\rm obl}_{14}-\bar{\bar{\Gamma}}^{\rm nor}_{14}}{j\eta_0\sin^2\theta_0} = \frac{\mu_{xx}}{\varepsilon_{xx}\mu_{xx}-\kappa^2_{xx}} \tag{19}$$

$$-\frac{\bar{\bar{\Gamma}}^{\rm obl}_{12}-\bar{\bar{\Gamma}}^{\rm nor}_{12}}{\sin^2\theta_0} = \frac{\kappa_{xx}}{\varepsilon_{xx}\mu_{xx}-\kappa^2_{xx}} \tag{20}$$

Notice that the left sides of these equations are completely known and are named N_1, N_2 and N_3, respectively. Therefore, the determination of $\varepsilon_{xx}, \mu_{xx}$ and κ_{xx} requires solving these equations which eventually leads to

$$(\varepsilon_{xx},\ \mu_{xx},\ \kappa_{xx}) = \frac{(N_1, N_2, N_3)}{N_1N_2-N_3^2}. \tag{21}$$

Briefly, the procedure of electromagnetic characterization from the knowledge of the transmission and reflection coefficients can be summarized as follows:

1) Solve (12)–(15) with considering oblique S-parameters along theorems 1 and 2 to find $\bar{\bar{\Phi}}^{\rm obl}$.
2) Solve (12)–(15) with assuming $\theta_0 = 0$ and considering normal S-parameters along theorems 1 and 2 to find $\bar{\bar{\Phi}}^{\rm nor}$.
3) Find $\bar{\bar{\Gamma}}$-matrices ($\bar{\bar{\Gamma}}^{\rm nor}$ and $\bar{\bar{\Gamma}}^{\rm obl}$) using (16).
4) Determine constitutive tensor parameters considering (4).

V. Numerical Example, Results and Discussion

To demonstrate the capability of the above approach, a bianisotropic specimen is considered. Here, due to the simplicity we use $\exp m$ and $\log m$ commands in MATLAB.

A. Dispersive Biaxial Bianisotropic Slab

Consider a biaxial bianisotropic slab with thickness $d = 1.6$ mm whose constitutive parameters have the following dispersion relations [17]:

$$\varepsilon = \varepsilon_b - \frac{\Omega_\varepsilon\,\omega_0^2}{\omega^2-\omega_0^2-j\omega\,\omega_0\delta_\varepsilon} \tag{22}$$

$$\mu = \mu_b - \frac{\Omega_\mu\,\omega^2}{\omega^2-\omega_0^2-j\omega\,\omega_0\,\delta_\mu} \tag{23}$$

$$\kappa = -\frac{\Omega_\kappa\,\omega_0\omega}{\omega^2-\omega_0^2-j\omega\,\omega_0\,\delta_\kappa} \tag{24}$$

where $\omega_0, \varepsilon_b, \mu_b, \delta_\varepsilon, \delta_\mu$ and δ_κ are resonant frequency, background relative permittivity and permeability, and damping factors, respectively. Assume that $\varepsilon_b = 4, \mu_b = 1, \omega_0 = 4$ GHz for all tensors elements, $\Omega_\varepsilon = 0.3, 1, 1.5$ and $\delta_\varepsilon = 0.02, 0.1, 0.05$ for $\varepsilon_{xx}, \varepsilon_{yy}$,

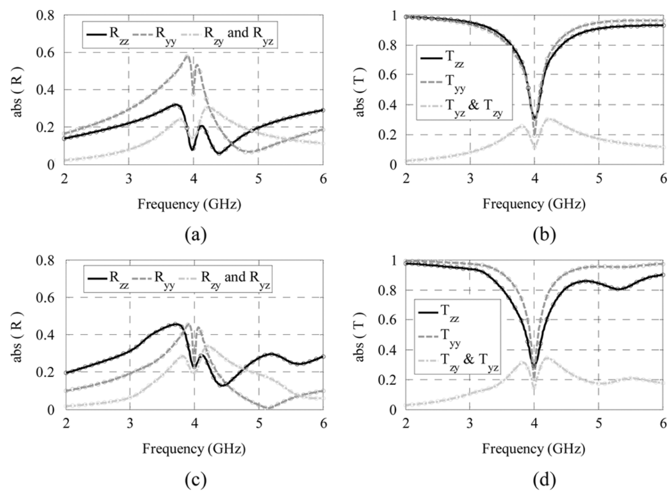

Fig. 2. Amplitudes of co- and cross-reflection and transmission coefficients of biaxial bianisotropic slab at (a) and (b) normal, and (c) and (d) oblique incident with $\theta_0 = 45°$. The results obtained by wave splitting method are shown with circles.

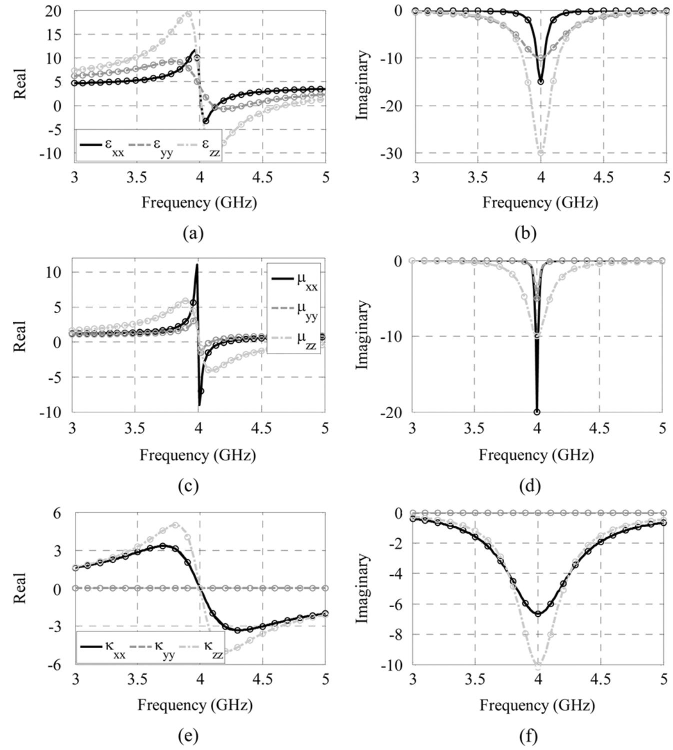

Fig. 3. Real and imaginary parts of constitutive parameters. (a) and (b) Relative permittivity tensor, (c) and (d) Relative permeability tensor, and (e) and (f) magneto-electric tensor. True values are shown with circles.

and ε_{zz}, respectively. Also, assume $\Omega_\mu = 0.1, 0.05, 0.5$ and $\delta_\mu = 0.005, 0.01, 0.05$ for μ_{xx}, μ_{yy}, and μ_{zz}, respectively. In addition, consider $\kappa_{yy} = 0$ and $\Omega_\kappa = 1, 1$, and $\delta_\mu = 0.15$ and 0.1 for κ_{xx} and κ_{zz}, respectively.

As an illustrative example, the scattering parameters of such a layer in the normal and oblique incidences at $\theta_0 = 0$ and 45° are calculated using the discussed method in Section II and are illustrated in Fig. 2. In order to verify the accuracy of results, the scattering parameters obtained by the wave splitting method discussed in [18] are also shown in this figure.

These calculated S-parameter data are fed into the extraction algorithm discussed in Section IV to obtain the constitutive tensor parameters. These obtained values are then compared to the known input values in Fig. 3. Observe that there is very good agreement between the true and retrieved values of constitutive parameters of bianisotropic slab. However, many example specimens with different material parameters and oblique illuminations were simulated and analyzed, but this case was chosen for discussion here due to its relevance to the bianisotropic material specimens.

B. Discussion

In conventional retrieval procedures, one should directly solve a set of nonlinear equations to obtain closed formulas for the constitutive electromagnetic parameters of the unknown slab in terms of scattering data. Most attempts at measuring the electromagnetic parameters at oblique incidence or accounting for bianisotropy have relied on using fully numerical optimization techniques. For instance, in uniaxial chiral composites, a set of 14 nonlinear equations was obtained, by solving which constitutive parameters of the slab was retrieved [11]. However, the bianisotropic media are far more complex and their characterization by the conventional full-wave methods requires solving a large number of nonlinear and complex equations that will certainly lead to the use of numerical optimizations techniques. It seems that this subject is the weak point of conventional retrieval procedures that is even much worse in more complex media. To sum up, the proposed retrieval procedure which is based on the state space approach and simple matrix manipulations allows avoiding nonlinearity and complexity of the problem.

VI. Summary and Conclusions

An analytic procedure for determining the electromagnetic constitutive tensor parameters of biaxial bianisotropic media based on the state space approach was presented. The proposed method allows avoiding nonlinearity and complexity of the problem, and is appropriate from computational and practical point of view. Material parameters extracted from this modeled data set were compared to the known material parameters in order to provide evidence relating to the convergence of the process.

Appendix

Based on Cayley-Hamilton theorem [19], for every 4×4 matrix X with distinct eigenvalues x_1, x_2, x_3 and x_4, there exist 4 scalar functions a_0, a_1, a_2, and a_3 for which

$$e^{\mathrm{X}} = \sum_{q=0}^{3} a_q \mathrm{X}^q. \tag{A1}$$

and

$$e^{x_p} = \sum_{q=0}^{3} a_q x_p^q, \quad \text{for } p = 1,\ 2,\ 3 \,\text{and}\, 4. \tag{A2}$$

By solving these four equations, unknown coefficients a_q are fully determined in terms of eigenvalues x_p [15]. For a biaxial bianisotropic layer, assuming $\bar{\bar{\mathrm{X}}} = -\bar{\bar{\Gamma}}_\omega d, g_1 = \bar{\bar{\mathrm{X}}}_{14}, g_2 = \bar{\bar{\mathrm{X}}}_{23}, g_3 = \bar{\bar{\mathrm{X}}}_{32}, g_4 = \bar{\bar{\mathrm{X}}}_{41}, g_5 = \bar{\bar{\mathrm{X}}}_{43}$, and $g_6 = \bar{\bar{\mathrm{X}}}_{12}$, the eigenvalues of $\bar{\bar{\mathrm{X}}}$ are

$$x_{\substack{1\\3}} = -x_{\substack{2\\4}} = \left\{ \frac{g_2 g_3 + g_1 g_4}{2} + g_5 g_6 \pm \left[\left(\frac{g_2 g_3 - g_1 g_4}{2} \right)^2 + g_5 g_6 (g_2 g_3 + g_1 g_4) + g_1 g_3 g_5^2 + g_2 g_4 g_6^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \tag{A3}$$

Then, after some matrix manipulations in (A1), the matrix exponential $\bar{\bar{\Phi}} = \exp(-\bar{\bar{\Gamma}}_\omega d)$ may be written as (9) where

$$\begin{cases} \Phi_{11} = a_0 + a_2(g_1g_4 + g_5g_6), \quad \Phi_{13} = a_2(g_1g_5 + g_2g_6) \\ \Phi_{22} = a_0 + a_2(g_2g_3 + g_5g_6), \quad \Phi_{31} = a_2(g_3g_5 + g_4g_6) \\ \Phi_{12} = a_3\left[g_1(g_3g_5 + g_4g_6) + g_6(g_2g_3 + g_5g_6)\right] + a_1g_6 \\ \Phi_{14} = a_3\left[g_1(g_1g_4 + g_5g_6) + g_6(g_1g_5 + g_2g_6)\right] + a_1g_1 \\ \Phi_{21} = a_3\left[g_2(g_3g_5 + g_4g_6) + g_5(g_1g_4 + g_5g_6)\right] + a_1g_5 \\ \Phi_{23} = a_3\left[g_2(g_2g_3 + g_5g_6) + g_5(g_1g_5 + g_2g_6)\right] + a_1g_2 \\ \Phi_{32} = a_3\left[g_3(g_2g_3 + g_5g_6) + g_6(g_4g_6 + g_3g_5)\right] + a_1g_3 \\ \Phi_{41} = a_3\left[g_4(g_1g_4 + g_5g_6) + g_5(g_3g_5 + g_4g_6)\right] + a_1g_4 \end{cases}. \tag{A4}$$

For the computation of the inverse matrix $\bar{\bar{\Phi}}^{-1}$, $\exp(\bar{\bar{\Gamma}}_\omega d)$ can be computed. It is evident that the eigenvalues remain unchanged rather than prior case. In this case, the following should be considered

$$e^{-\mathrm{X}} = \sum_{q=0}^{3} a'_q(-\mathrm{X})^q. \tag{A5}$$

It can be easily seen that

$$a'_0 = a_0, \quad a'_1 = -a_1, \quad a'_2 = a_2, \quad a'_3 = -a_3. \tag{A6}$$

Therefore, after some straightforward matrix manipulations, one can obtain $\bar{\bar{\Phi}}^{-1}$ as presented in (10).

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