J. of Electromagn. Waves and Appl., Vol. 26, 875–884, 2012

ELECTROMAGNETIC SCATTERING FROM INHOMOGENEOUS PLANAR LAYERED MEDIA USING NOTATION OF PROPAGATORS

V. Nayyeri, D. Zarifi*, and M. Soleimani

Antenna Research Laboratory, Iran University of Science and Technology, Tehran 1684613114, Iran

Abstract—In this paper, the reflection and transmission of an electromagnetic plane wave from (through) planar layered of inhomogeneous bi-anisotropic media are considered. Inhomogeneous layers are decomposed into thin homogeneous sub-layers, and the method of propagators is used for analysis of the wave interaction with planar multi-layered of homogenous bi-anisotropic media. The most interesting property of the presented method is its systematic approach. Finally, the validity of the method is verified by considering some special types of bi-anisotropic media and comparing the obtained results from the presented method with the other reported methods.

1. INTRODUCTION

There has been increasing interest in studying interaction of electromagnetic (EM) fields with bi-anisotropic media over the years [1–5]. Due to anisotropy of a bi-anisotropic medium, its constitutive relations can be written in a dyadic form, i.e., $\boldsymbol{D} = \varepsilon_0 \boldsymbol{\upvarepsilon} \cdot \boldsymbol{E} + \sqrt{(\varepsilon_0\mu_0)}\boldsymbol{\upxi} \cdot \boldsymbol{H}$ and $\boldsymbol{B} = \mu_0 \boldsymbol{\upmu} \cdot \boldsymbol{H} + \sqrt{(\varepsilon_0\mu_0)}\boldsymbol{\upzeta} \cdot \boldsymbol{E}$, where ε_0 and μ_0 are the permittivity and permeability of vacuum, $\boldsymbol{\upvarepsilon}$ and $\boldsymbol{\upmu}$ are the relative permittivity and permeability dyadics, and $\boldsymbol{\upxi}$ and $\boldsymbol{\upzeta}$ are magneto-electric coupling dyadics [1]. To simplify the notations, *scalars*, ***vectors***, and **dyadics** are represented by *italic*, ***italic bold***, and **non-italic bold** letters, respectively. Based on these constructive relations, bi-anisotropic media, incorporating large variety of media, such as anisotropic media, gyrotropic media, chiral and bi-isotropic media, have been found to have various applications in microwave and antenna engineering [6–15].

Received 8 February 2012, Accepted 8 March 2012, Scheduled 19 March 2012
* Corresponding author: Davoud Zarifi (zarifi@iust.ac.ir).

EM scattering from planar layered media is an important and basic problem in the electromagnetics. Although analysis of scattering from stratified inhomogeneous media is much more complicated than that from homogeneous media, the scattering from planar layers of simple isotropic inhomogeneous media has been intensively investigated and several approaches have been presented such as Richmond method [16], solving Riccati equation [17], full-wave analysis [18, 19], finite-difference method [20], using Taylor's series expansion [21], using Fourier series expansion [22], method of moments [23], equivalent source method [24], and cascading thin linear layers method [25]. On the other hand, the scattering from planar layers of anisotropic inhomogeneous media is investigated in [26, 27]. Furthermore, the analysis of wave interaction with an inhomogeneous chiral slab is recently reported in [28] using Taylor's series expansion.

A very effective and systematic approach for solving scattering from stratified bi-anisotropic media has been discussed in [29]. In this study, we use this method for analyzing scattering from planar layers of inhomogeneous bi-anisotropic media including all types of linear media. In this method the notation of propagators which is a strong tool for solving the scattering properties of a stratified medium is applied. The propagators propagate the total tangential electric and magnetic fields in the layers and only outside the slab do the up- and down-going parts of the fields need to be identified. In Section 2, fundamental relations of the method of propagators, introduced in [29] are briefly presented. Then in Section 3, applicability of this method for inhomogeneous media is shown through some examples. In order to achieve the validity, two special cases are considered and the obtained results from this method are compared with the previously reported results.

2. FUNDAMENTAL RELATIONS OF THE METHOD OF PROPAGATORS

In Fig. 1, a laterally homogenous stratified medium, composed of N layers of bianisotropic media is considered. Analysis of scattering from a laterally homogenous medium can be performed through the following steps [29].

First, the electric and magnetic fields [$\boldsymbol{E}(\boldsymbol{r}, \omega)$ and $\boldsymbol{H}(\boldsymbol{r}, \omega)$] are transferred in to the spectral domain using the Fourier transform (FT) by:

$$\boldsymbol{A}\left(z, \boldsymbol{k_t}, \omega\right) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \boldsymbol{A}\left(\boldsymbol{r}, \omega\right) \exp\left\{-j\boldsymbol{k_t}\cdot\boldsymbol{\rho}\right\} dx\, dy \qquad (1)$$

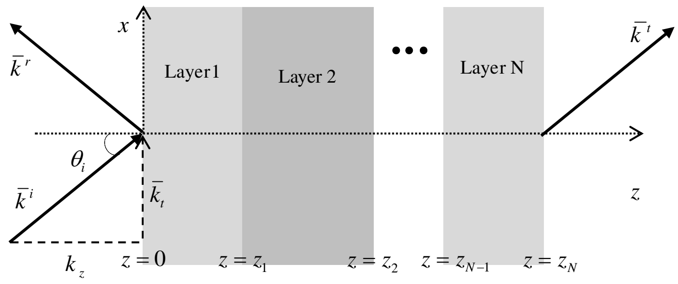


Figure 1. Plane wave incidence upon a stratified structure of N layers.

where $\boldsymbol{\rho} = \hat{\boldsymbol{x}}x + \hat{\boldsymbol{y}}y$ is the radius vector in the xy-plane, and $\boldsymbol{k_t} = \hat{\boldsymbol{x}}k_x + \hat{\boldsymbol{y}}k_y$ is the tangential wave vector, and $\boldsymbol{A}$ is either $\boldsymbol{E}$ or $\boldsymbol{H}$.

Second, the FT of the electric and magnetic fields are decomposed into tangential and normal components. For instance, for the electric field we have $\boldsymbol{E}(z, \boldsymbol{k_t}, \omega) = \boldsymbol{E}_{xy}(z) + \hat{\boldsymbol{z}}E_z(z)$.

Third, substituting the constitutive relations into the Maxwell's equations and assuming $\exp(-j\omega t)$ time convention, the following system of differential equations in terms of the tangential components of the fields is obtained

$$\frac{d}{dz}\begin{bmatrix}\boldsymbol{E_{xy}}(z)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z)\end{bmatrix} = jk_0\mathbf{M}(z)\cdot\begin{bmatrix}\boldsymbol{E_{xy}}(z)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z)\end{bmatrix} \tag{2}$$

where k_0 is the wave number in the free-space, η_0 the free-space intrinsic impedance, $\mathbf{J} = -\hat{\boldsymbol{x}}\hat{\boldsymbol{y}} + \hat{\boldsymbol{y}}\hat{\boldsymbol{x}}$ the 2-D rotation dyadic, and $\mathbf{M} = \begin{bmatrix}\mathbf{M_{11}} & \mathbf{M_{12}}\\ \mathbf{M_{21}} & \mathbf{M_{22}}\end{bmatrix}$ the fundamental dyadic of the medium. Elements of the fundamental dyadic $\mathbf{M}_{ij}$, given in [29] are 2-D dyadics, depending on the tangential wave vector $\boldsymbol{k_t}$, and the constitutive dyadics of the medium. Using the above analysis, the formal solution of (2) for an observation point inside layer n (i.e., $z_{n-1} \le z \le z_n$) is

$$\begin{bmatrix}\boldsymbol{E_{xy}}(z)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z)\end{bmatrix} = S\exp\left\{jk_0\int_{z_{n-1}}^{z_n}\mathbf{M_n}(z')dz'\right\}\cdot\begin{bmatrix}\boldsymbol{E_{xy}}(z_{n-1})\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z_{n-1})\end{bmatrix} \tag{3}$$

where $\mathbf{M_n}(z)$ is the fundamental dyadic computed for the constitutive dyadics of layer n and S the spatial ordering operator. Due to the continuity of tangential fields, their values at the left-hand side boundary ($z = 0$) can be simply mapped to the right-hand side boundary ($z > 0$). Therefore, at the boundary ($z = z_N$) we have

$$\begin{bmatrix}\boldsymbol{E_{xy}}(z_N)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z_N)\end{bmatrix} = \mathbf{P}\cdot\begin{bmatrix}\boldsymbol{E_{xy}}(z=0)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z=0)\end{bmatrix} \tag{4}$$

where $\mathbf{P}$ is the propagator dyadic of the stratified media. In the case of layered structure of homogeneous media the propagator dyadic can be simplified to

$$\mathbf{P}=\begin{bmatrix}\mathbf{P_{11}} & \mathbf{P_{12}}\\ \mathbf{P_{21}} & \mathbf{P_{22}}\end{bmatrix}=e^{(jk_0(z_N-z_{N-1})\mathbf{M_N})}\cdot\ldots\cdot e^{(jk_0(z_2-z_1)\mathbf{M_2})}\cdot e^{(jk_0z_1\mathbf{M_1})} \quad (5)$$

and can be easily calculated using MATLAB *expm* command. In (5), it is assumed that each layer is a homogeneous medium. On the other hand, in the case of an inhomogeneous layer, the layer can be decomposed into thin homogeneous sub-layers. Then considering (5), the propagator dyadic of the layer is determined by multiplication of the propagator dyadics of its sub-layers.

Fourth, assuming that the structure is illuminated by an obliquely incident plane wave (see Fig. 1), the electric and magnetic fields in free-space region, $z<0$, are the sum of incident ($\boldsymbol{E}^i$, $\boldsymbol{H}^i$) and reflected ($\boldsymbol{E}^r$, $\boldsymbol{H}^r$) plane waves. Considering that incident and reflected fields are decomposed into the tangential and normal components, i.e., $\boldsymbol{E}^i=\boldsymbol{E}^i_{\boldsymbol{xy}}+\hat{\boldsymbol{z}}E^i_z$ and $\boldsymbol{E}^r=\boldsymbol{E}^r_{\boldsymbol{xy}}+\hat{\boldsymbol{z}}E^r_z$, the wave splitting technique allows explaining the tangential components of the electric and magnetic fields at $z=0$ by that of the forwarding (incident) and backwarding (reflected) electric fields as:

$$\begin{bmatrix}\boldsymbol{E_{xy}}(0)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(0)\end{bmatrix}=\begin{bmatrix}\mathbf{I_t} & \mathbf{I_t}\\ -\mathbf{O}^{-1} & \mathbf{O}^{-1}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{E}^i_{\boldsymbol{xy}}(0)\\ \boldsymbol{E}^r_{\boldsymbol{xy}}(0)\end{bmatrix} \quad (6)$$

where $\mathbf{I_t}=\hat{\boldsymbol{x}}\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}\hat{\boldsymbol{y}}$ is the 2-D identity dyadic, $\mathbf{O}^{-1}=k_0/k_z[\mathbf{I_t}+1/k_0^2\boldsymbol{k_t}\times(\boldsymbol{k_t}\times\mathbf{I_t})]$, and $\boldsymbol{k_t}$ and k_z are the tangential and normal components of the incident and reflected wave vectors, i.e., $\boldsymbol{k}^i=\boldsymbol{k_t}+\hat{\boldsymbol{z}}k_z$ and $\boldsymbol{k}^r=\boldsymbol{k_t}-\hat{\boldsymbol{z}}k_z$. On the other hand, the fields at boundary $z=z_N$ is forwarding (transmitted), thus by applying the wave splitting technique we have:

$$\begin{bmatrix}\boldsymbol{E_{xy}}(z_N)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z_N)\end{bmatrix}=\begin{bmatrix}\mathbf{I_t} & \mathbf{I_t}\\ -\mathbf{O}^{-1} & \mathbf{O}^{-1}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{E}^t_{\boldsymbol{xy}}(z_N)\\ 0\end{bmatrix} \quad (7)$$

where $\boldsymbol{E}^t_{\boldsymbol{xy}}(z_N)$ is the transmitted field at $z=z_N$. The inverse relation can be written as

$$\begin{aligned}\begin{bmatrix}\boldsymbol{E}^t_{\boldsymbol{xy}}(z_N)\\ 0\end{bmatrix}&=\begin{bmatrix}\mathbf{I_t} & \mathbf{I_t}\\ -\mathbf{O}^{-1} & \mathbf{O}^{-1}\end{bmatrix}^{-1}\cdot\begin{bmatrix}\boldsymbol{E_{xy}}(z_N)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z_N)\end{bmatrix}\\ &=\frac{1}{2}\begin{bmatrix}\mathbf{I_t} & -\mathbf{O}\\ \mathbf{I_t} & \mathbf{O}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{E_{xy}}(z_N)\\ \eta_0\mathbf{J}\cdot\boldsymbol{H_{xy}}(z_N)\end{bmatrix}. \end{aligned} \quad (8)$$

Substituting (6) in (4) and the obtained relation in (8) we obtain:

$$\begin{bmatrix}\boldsymbol{E}^t_{\boldsymbol{xy}}(z_N)\\ 0\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\mathbf{I_t} & -\mathbf{O}\\ \mathbf{I_t} & \mathbf{O}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{P_{11}} & \mathbf{P_{12}}\\ \mathbf{P_{21}} & \mathbf{P_{22}}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{I_t} & \mathbf{I_t}\\ -\mathbf{O}^{-1} & \mathbf{O}^{-1}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{E}^i_{\boldsymbol{xy}}(0)\\ \boldsymbol{E}^r_{\boldsymbol{xy}}(0)\end{bmatrix} \quad (9)$$

Fifth, the transverse reflection and transmission dyadics $\mathbf{r}_{\boldsymbol{xy}}$ and $\mathbf{t}_{\boldsymbol{xy}}$, whose multiplications by the transverse incident field reveal the transverse reflected and transmitted fields, are defined by

$$\boldsymbol{E}^{r}_{\boldsymbol{xy}}(0) = \mathbf{r_{xy}} \cdot \boldsymbol{E}^{i}_{\boldsymbol{xy}}(0), \quad \boldsymbol{E}^{t}_{\boldsymbol{xy}}(z_N) = \mathbf{t_{xy}} \cdot \boldsymbol{E}^{i}_{\boldsymbol{xy}}(0) \tag{10}$$

and can be obtained by first substituting (10) in (9), i.e.,

$$\begin{bmatrix} \mathbf{t_{xy}} \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \mathbf{I_t} & -\mathbf{O} \\ \mathbf{I_t} & \mathbf{O} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{P_{11}} & \mathbf{P_{12}} \\ \mathbf{P_{21}} & \mathbf{P_{22}} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I_t} & \mathbf{I_t} \\ -\mathbf{O}^{-1} & \mathbf{O}^{-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I_t} \\ \mathbf{r_{xy}} \end{bmatrix} \tag{11}$$

and then solving the system of the obtained equations. The result is:

$$\mathbf{r_{xy}} = -\mathbf{T_{22}^{-1}} \cdot \mathbf{T_{21}}, \quad \mathbf{t_{xy}} = \mathbf{T_{11}} + \mathbf{T_{12}} \cdot \mathbf{r_{xy}} \tag{12}$$

where $\begin{bmatrix} \mathbf{T_{11}} & \mathbf{T_{12}} \\ \mathbf{T_{21}} & \mathbf{T_{22}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \mathbf{I_t} & -\mathbf{O} \\ \mathbf{I_t} & \mathbf{O} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{P_{11}} & \mathbf{P_{12}} \\ \mathbf{P_{21}} & \mathbf{P_{22}} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I_t} & \mathbf{I_t} \\ -\mathbf{O}^{-1} & \mathbf{O}^{-1} \end{bmatrix}$.

3. APPLICABILITY OF THE METHOD OF PROPAGATORS FOR ANALYZING SCATTERING FROM PLANAR LAYERS OF INHOMOGENEOUS MEDIA

In this section, three examples are provided to illustrate the applicability of the method of propagators for analyzing scattering from planar layers of inhomogeneous media. In the proposed method, each layer is decomposed into 100 homogenous sub-layers and then the method of propagators is applied to determine the reflection and transmission dyadics. Although in general the proposed method allows considering planar layers of bianisotropic inhomogeneous media, in the first two examples, special cases are considered in order to compare the obtained results with those of previously published methods and achieve the validity of the presented method.

The most valuable and interesting property of the presented method is its systematic approach, allowing one can easily apply this method on any types of planar layers of inhomogeneous media and simply implement it in a programming language supporting matrix manipulations such as MATLAB. In addition the consumed time for this method is much less than that for numerical methods such as finite-difference technique. For instance, the time consumed for three examples presented in this paper was less than 5 seconds using a computer with Intel Core(TM) I5 CPU and MATLAB program.

3.1. Example 1 (Two Layers of Homogeneous Anisotropic Media)

In the first example a two layered slab of homogeneous anisotropic media with thicknesses of $d_1 = 4\,\text{mm}$ and $d_2 = 1\,\text{mm}$ which constitutive dyadics of the layers are given by $\boldsymbol{\varepsilon}_1 = 5\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + 4\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + 7\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$, $\boldsymbol{\mu}_1 = 7\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + 12\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + 30\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$, $\boldsymbol{\varepsilon}_2 = -1\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + (-1.8)\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + (-1.4)\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$, and $\boldsymbol{\mu}_2 = (-1.4)\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + (-2.4)\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + (-6)\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$ is considered. The magneto-electric coupling dyadics of the media are considered to be zero. The slab is illuminated by an oblique incidence of a linearly polarized plane wave (TE^z or TM^z) with unity amplitude, and the excitation frequency of 20 GHz. The problem of plane wave propagation through the same two-layered slab was discussed in [30]. The amplitudes of co- and cross-reflection coefficients obtained from the presented method and the method of [30] versus the angle of incidence are compared in Fig. 2. It can be seen that the obtained results from the presented method are in an excellent agreement with those from [30].

3.2. Example 2 (Inhomogeneous Chiral Slab)

In the second example, the scattering of a plane wave from an infinite inhomogeneous chiral slab is considered. The constitutive equations for a chiral medium assuming a time harmonic dependence $\exp(-j\omega t)$ are given by $\boldsymbol{D} = \varepsilon_r\varepsilon_0\boldsymbol{E} + j\kappa\sqrt{(\varepsilon_0\mu_0)}\boldsymbol{H}$ and $\boldsymbol{B} = \mu_r\mu_0\boldsymbol{H} - j\kappa\sqrt{(\varepsilon_0 mu_0)}$ $\boldsymbol{E}$. Assume a plane wave with TE^z polarization, unity amplitude, and the frequency of 1 GHz obliquely illuminates an inhomogeneous chiral slab with thickness of 0.2 m whose constitutive parameters have the profiles of $\varepsilon_r(z) = 4z$, $\mu_r(z) = 1$, and $\kappa(z) = e^{2z}$. The amplitudes of co- and cross-reflection and co-transmission coefficients obtained from the proposed method versus the angle of incidence are shown in Figs. 3(a) and 3(b), respectively. In order to verify the accuracy, the obtained results from the Taylor's series expansion method discussed in [28] are also illustrated in Figs. 3(a) and 3(b). Apparently, there is an excellent agreement between the results of the two different methods.

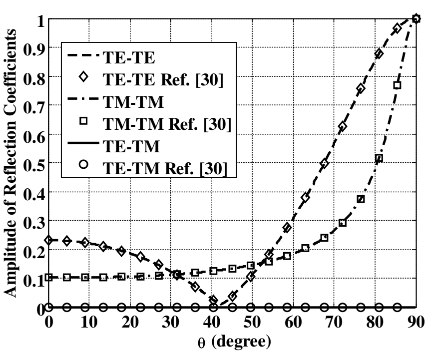
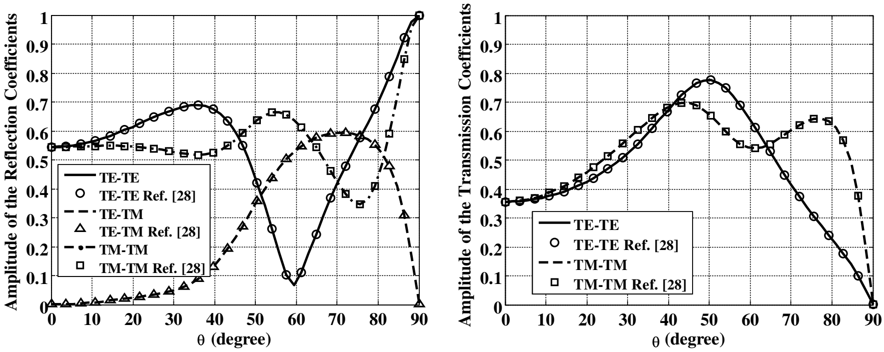
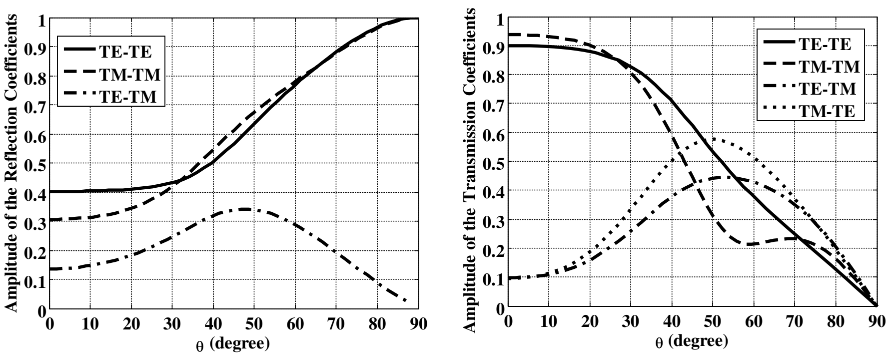


Figure 2. Co- and cross-reflection coefficients as functions of incident angle θ for a two-layered slab of homogeneous anisotropic media.



(a) (b)

Figure 3. Reflection and transmission coefficients as functions of incident angle θ for an inhomogeneous chiral slab. (a) Co- and cross-reflection coefficients. (b) Co-transmission coefficients.



(a) (b)

Figure 4. Co- and cross components of reflection (a) and transmission (b) dyadic as functions of incident angle θ for an inhomogeneous bianisotropic slab.

It should be noticed that although the existence of the derivatives of the parameters profiles with respect to the z variable is a necessary condition for Taylor's series expansion method, the presented method is not restricted to this constraint.

3.3. Example 3 (Inhomogeneous Bi-anisotropic Layer)

In the prior examples, the validity of the presented method was verified. In the third example, the problem of scattering from an inhomogeneous bi-anisotropic layer with thicknesses of 0.2 m and constitutive dyadics given by $\boldsymbol{\varepsilon}(z) = \exp(z/d)\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \exp(2z/d)\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + \exp(3z/d)\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$, $\boldsymbol{\mu}(z) =$

$\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + \hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$, and $\boldsymbol{\xi}(z) = -\boldsymbol{\zeta}(z) = j[(z/d)\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + (2z/d)\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + (3z/d)\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}]$ is considered. The layer is illuminated by an oblique incidence of a linearly polarized plane wave (TE^z or TM^z) with unity amplitude, and the excitation frequency of 1 GHz. The amplitudes of the co- and cross-components of reflection and transmission dyadics, obtained from the presented method versus the angle of incidence are shown in Fig. 4.

Finally, according the discussed examples, it can be concluded that the proposed method is efficiently applicable for analyzing scattering from stratified inhomogeneous media.

4. CONCLUSIONS

This paper presents analysis of electromagnetic scattering from planar layered structure of inhomogeneous bianisotropic media. Inhomogeneous layers are decomposed into thin homogeneous sub-layers, and the method of propagator which has been proposed for wave interaction with stratified bianisotropic media applied. Since the presented approach is very systematic, it can be simply implemented in programming languages supporting matrix manipulations such as MATLAB. The validity of the presented method as well as the correctness of our code is achieved by providing some numerical examples and comparing the obtained results results with the results of previously published works for two special cases.

REFERENCES

1. Serdyukov, A., I. Semchenko, S. Tretyakov, and A. Sihvola, *Electromagnetics of Bi-anisotropic Materials: Theory and Applications*, Taylor & Francis, 2001.
2. Lindell, I. V. and F. Olyslager, "Potentials in bi-anisotropic media," *Journal of Electromagnetic Waves and Applications*, Vol. 15, No. 1, 3–18, 2001.
3. Lindell, I. V., "Affine transformations and bi-anisotropic media in differential-form approach," *Journal of Electromagnetic Waves and Applications*, Vol. 18, No. 9, 1259–1273, 2004.
4. Sheen, J., "Electromagnetic fields in a biaxial anisotropic medium," *Journal of Electromagnetic Waves and Applications*, Vol. 19, No. 6, 753–767, 2005.
5. Nayyeri, V., M. Soleimani, J. Rashed-Mohassel, and M. Dehmollaian, "FDTD modeling of dispersive bianisotropic media using Z-transform," *IEEE Trans. on Antennas and Propagat.*, Vol. 59, No. 6, 2268–2279, 2011.

6. Zarifi, D., H. Oraizi, and M. Soleimani, "Improved performance of circularly polarized antenna using semi-planar chiral metamaterial covers," *Progress In Electromagnetics Research*, Vol. 123, 337–354, 2012.
7. Zarifi, M. S. and V. Nayyeri, "A novel dual-band chiral metamaterial structure with giant optical activity and negative refractive index," *Journal of Electromagnetic Waves and Applications*, Vol. 56, No. 2, 251–263, 2012.
8. Ardakani, H. H., S. Fallahzadeh, and J. Rashed-Mohassel, "Phase velocities equalization of coupled microstrip lines using Ω-shaped particles and suppression of the second harmonic," *IEEE Trans. on Microwave Theory and Tech.*, Vol. 60, No. 3, 464–470, 2012.
9. Naqvi, A., A. Hussain, and Q. A. Naqvi, "Waves in fractional dual planar waveguides containing chiral nihility metamaterial," *Journal of Electromagnetic Waves and Applications*, Vol. 24, Nos. 11–12, 1575–1586, 2010.
10. Qamar, S. R., A. Naqvi, A. A. Syed, and Q. A. Naqvi, "Radiation characteristics of elementary sources located in unbounded chiral nihility metamaterial," *Journal of Electromagnetic Waves and Applications*, Vol. 25, Nos. 5–6, 713–722, 2011.
11. Dong, J., J. Li, and F.-Q. Yang, "Guided modes in the four-layer slab waveguide containing chiral nihility core," *Progress In Electromagnetics Research*, Vol. 112, 241–255, 2011.
12. Wu, Z., B. Q. Zhang, and S. Zhong, "A double-layer chiral metamaterial with negative index," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 7, 983–992, 2010.
13. Li, Z., F.-Q. Yang, and J. Dong, "Design and simulation of L-shaped chiral negative refractive index structure," *Progress In Electromagnetics Research*, Vol. 116, 395–408, 2011.
14. Ghaffar, A., "Focusing by a cylindrical plano convex lens of chiral medium," *Journal of Electromagnetic Waves and Applications*, Vol. 25, Nos. 5–6, 833–844, 2011.
15. Dong, W., L. Gao, and C.-W. Qiu, "Goos-Hänchen shift at the surface of chiral negative refractive media," *Progress In Electromagnetics Research*, Vol. 90, 255–268, 2009.
16. Richmond, J. H., "Transmission through inhomogeneous plane layers," *IRE Trans. on Antennas and Propag.*, 300–305, May 1962.
17. Chew, W. C., *Waves and Fields in Inhomogeneous Media*, IEEE Press, New York, 1990.
18. Urbani, F., L. Vegni, and A. Toscano, "Inhomogeneous layered planar structures: an analysis of the reflection coefficients," *IEEE*

Trans. on Magn., 2771–2774, Sep. 1998.

19. Rojas, J. A. M., J. Alpuente, J. Piñeiro, and R. Sanchez-Montero, "Rigorous full vectorial analysis of electromagnetic wave propagation in 1D," *Progress In Electromagnetics Research*, Vol. 63, 89–105, 2006.
20. Khalaj-Amirhosseini, M., "Analysis of lossy inhomogeneous planar layers using finite difference method," *Progress In Electromagnetics Research*, Vol. 59, 187–198, 2006.
21. Khalaj-Amirhosseini, M., "Analysis of lossy inhomogeneous planar layers using Taylor's series expansion," *IEEE Trans. on Antennas and Propagat.*, Vol. 54, No. 1, 130–135, Jan. 2006.
22. Khalaj-Amirhosseini, M., "Analysis of lossy inhomogeneous planar layers using fourier series expansion," *IEEE Trans. on Antennas and Propagat.*, Vol. 55, No. 2, 489–493, Feb. 2007.
23. Khalaj-Amirhosseini, M., "Analysis of lossy inhomogeneous planar layers using the method of moments," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 14, 1925–1937, 2007.
24. Khalaj-Amirhosseini, M., "Analysis of lossy inhomogeneous planar layers using equivalent sources method," *Progress In Electromagnetics Research*, Vol. 72, 61–73, 2007.
25. Khalaj-Amirhosseini, M., "To analyze inhomogeneous planar layers by cascading thin linear layers," *Progress In Electromagnetics Research B*, Vol. 3, 95–104, 2008.
26. Morgan, M. A., D. L. Fisher, and E. A. Milne, "Electromagnetic scattering by stratified inhomogeneous anisotropic media," *IEEE Trans. on Antennas and Propagat.*, Vol. 35, No. 2, 191–197, 1987.
27. Bass, F. and L. Resnick, "The electromagnetic-wave propagation through a stratified inhomogeneous anisotropic medium," *Progress In Electromagnetics Research*, Vol. 48, 67–83, 2004.
28. Zarifi, D., A. Abdolali, M. Soleimani, and V. Nayyeri, "Inhomogeneous planar layered chiral media: Analysis of wave propagation and scattering using Taylor's series expansion," *Progress In Electromagnetics Research*, Vol. 125, 119–135, 2012.
29. Rikte, S., G. Kristensson, and M. Anderson, "Propagation in bianisotropic media-reflection and transmission," *IEEE Proc. Microw. Antennas Propag.*, Vol. 148, No. 1, Feb. 2001.
30. Mirzaei, H. and J. Rashed-Mohassel, "Zero reflection from anisotropic metamaterial stratified structures," *Progress In Electromagnetics Research B*, Vol. 26, 321–334, 2010.