A decentralized adaptive robust method for chaos control

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This paper presents a control strategy, which is based on sliding mode control, adaptive control, and fuzzy logic system for controlling the chaotic dynamics. We consider this control paradigm in chaotic systems where the equations of motion are not known. The proposed control strategy is robust against the external noise disturbance and system parameter variations and can be used to convert the chaotic orbits not only to the desired periodic ones but also to any desired chaotic motions. Simulation results of controlling some typical higher order chaotic systems demonstrate the effectiveness of the proposed control method. © *2009 American Institute of Physics*.
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During the last several years, several control strategies have been proposed for chaos control, but developing a control strategy which provides satisfactory tracking performance, to be robust against external noise disturbance and time varying system parameters, to be able to convert the undesired chaotic motion to any desired motion without requiring the equation of the motion, is still an open problem. In this paper, we propose a control strategy, which is based on synergistic combination of adaptive control with sliding mode control (SMC) for chaos control. The main advantage of SMC derives from the property of robustness to system uncertainties and external disturbances. However, the main drawback of the standard sliding modes is mostly related to the so-called chattering caused by the high-frequency control switching. In order to limit the chattering phenomena and to preserve the main advantages of the original SMC, we propose a new SMC by combining an adaptive nonlinear compensator with SMC and is referred to as adaptive robust control (ARC). For controlling chaos in high dimensions, we design a set of independent adaptive robust controllers. Each state variable of chaotic system has its own controller. The results on the Duffing's equation and Lorenz system show that the controller provided accurate converting of unwanted chaotic motions to any desired motions with fast convergence during time-varying system parameters and external noise disturbances.

I. INTRODUCTION

Now, it has been well known that the chaotic dynamics exist in a large variety of natural systems (e.g., biological and physical systems). Motivated by potential applications in physics, biological engineering, information processing, and communication theory, control of chaotic dynamics has received an increasing interest. The first model-free chaos control method, known as the OGY method, was presented by Ott, Grebogi, and Yorke to stabilize one of the unstable periodic orbits (UPOs) by perturbing an accessible system parameter over.[1,2] The method requires the location of the desired periodic orbit that is determined via a long time series and the linearized dynamics about the periodic orbit. The control is activated only when the trajectory enters a small neighborhood of the desired UPO so one has to wait for some time for this to occur if the trajectory starts from a randomly chosen initial condition in the basin of attraction of the chaotic attractor.[3] Even so, the controller may not be able to bring a trajectory that is already in the neighborhood of the desired UPO to the vicinity of the periodic orbit. In addition, the OGY method for controlling higher dimensional chaotic systems is quite difficult.[4] Its difficulty lies in the situation where the system Jacobian at a fixed point has complex eigenvalues or multiple unstable eigenvalues. Due to this fact, Yu *et al.*[2] extended the OGY chaos control to be useful for controlling higher order chaotic systems,[4] especially in the case where some of the eigenvalues of the system Jacobian are complex conjugates. The method relies on the identification of suitable invariant manifolds using a locally linear model and it was shown that the resulting dynamics is asymptotically stable.

Moreover, a number of chaos control strategies have been proposed, which are based on feedback control system[5–8] as well as SMC.[9–12] However, these methods are applicable for a specific chaotic system and, in contrast to the OGY control, require the full knowledge on system dynamics. Moreover, these methods only guarantee the asymptotic stability.[6–11] Asymptotic stability implies that the system trajectories converge to the equilibrium as time goes to infinity.

Traditionally, the goal of chaos control has been to stabilize one of the UPOs.[4] However, for some applications, it is desirable to set the system to behave in a chaotic manner, to convert a stable equilibrium point, periodic orbit, or an unwanted chaotic behavior to a desired chaotic motion with prescribed properties. For example, there is abundant evidence that supports the existence of chaos at all levels from the simplest to the most complex forms of organization of the nervous system.[13,14] The nonlinear dynamics analysis of

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human electroencephalogram (EEG) has shown the existence of chaotic attractor[15–18] and changes in the attractor dimension during sleep stages and diseases.[19–21] There is a growing agreement that some diseases such as epileptic seizure and depression are related to a loss of the complexity of brain signals.[22,23] It is commonly believed that seizure episodes are characterized by bifurcations to system states of low complexity.[23,24] Therefore, one can imagine the treatment of biological disorders by converting an unwanted chaotic dynamics to a desired chaotic motion (i.e., controlling the complexity).

The aim of this work is to develop a chaos control strategy that does not require the equations of motion to be known, to be general so that it can be applied to any chaotic systems, to be robust against external disturbances and noise, and to be applicable in real-time applications. Another important issue in designing a chaos control strategy for practical applications is the existence of various types of uncertainties (e.g., system uncertainty, unmodeled dynamics, and unknown exogenous disturbances). The dynamics of a chaotic plant are highly time varying and nonlinear. A useful and powerful control scheme to deal with the uncertainties, nonlinearities, and external disturbances is the SMC.[25] Nevertheless, the SMC suffers from the high-frequency oscillations in the control input, which is called "chattering."[26] Chattering is undesirable because it can excite unmodeled and high-frequency plant dynamics. A simple method for alleviation of chattering is using a suitable boundary layer around the sliding surface, in which the switching function is approximated by a linear feedback gain when the state trajectory lies within the boundary layer.[25–27] Within the boundary layer, the system no longer behaves as dictated by SMC. By introducing boundary layer, chattering can be reduced, but tracking performance and robustness are compromised.

In order to limit the chattering phenomena and to preserve the main advantages of the original SMC, we propose a control strategy, which is based on SMC, and adaptive control, referred to as ARC, for controlling the chaos dynamics.

One way to control chaos is to make parameter perturbations to an accessible system parameter.[1] However, when the system parameters are not accessible or cannot be changed easily, these methods cannot be used. To solve this problem, some researchers introduced an additive input to the chaotic systems.[28,29] Controlling the complexity using additive inputs may need to control the entire set of state variables. In this case, we will encounter the control of a multi-input multi-output (MIMO) nonlinear dynamical system. The controller complexity of a MIMO system can be considerably reduced if decentralized control schemes are used. The decentralized control problem is to design a set of independent controllers in which each subsystem is controlled by a stand-alone controller.[30] Each controller, developed based only on local information and measurements, operates solely on its associated subsystem. The interaction between the subsystems is taken as external disturbances for each isolated subsystem.

In this work, we present a general decentralized ARC strategy, which is based on ARC for online controlling the complexity of the chaotic systems.

II. BRIEF INTRODUCTION OF SLIDING MODE CONTROL

Consider the following nonlinear system:

$$\ddot{x} = f(\mathbf{x},t) + g(\mathbf{x},t)\cdot u(t) + d(t), \tag{1}$$

where $x(t)$ is the state to be controlled so that it follows a desired trajectory $x_d(t)$, $d(t)$ is the external disturbances, which is unknown but bounded by the known function, i.e., $|d(t)| \le D$, and $u(t)$ is the control input. The nonlinear dynamics $f(\mathbf{x},t)$ and control gain $g(\mathbf{x},t)$ are not known exactly but are estimated as the known nominal dynamics $\hat{f}(\mathbf{x},t)$ and $\hat{g}(\mathbf{x},t)$, respectively, with the bounded estimation errors. With uncertainties, the dynamic equation of the system (1) can be modified as

$$\begin{aligned}\ddot{x} &= (f(\mathbf{x},t) + \Delta f(\mathbf{x},t)) + (g(\mathbf{x},t) + \Delta g(\mathbf{x},t))\cdot u(t) + d(t)\\ &= f(\mathbf{x},t) + g(\mathbf{x},t)\cdot u(t) + w(\mathbf{x},t),\end{aligned} \tag{2}$$

where $\Delta f(\mathbf{x},t)$ and $\Delta g(\mathbf{x},t)$ denote unmodeled dynamics and parameter uncertainties; $w(\mathbf{x},t)$ is the lumped uncertainty and defined as

$$w(\mathbf{x},t) = \Delta f(\mathbf{x},t) + \Delta g(\mathbf{x},t)u(t) + d(t). \tag{3}$$

Here the bound of the lumped uncertainty is assumed to be given; that is,

$$|w(\mathbf{x},t)| < \upsilon. \tag{4}$$

The objective of the controller is to design a control law to force the system state vector to track a desired state vector in the presence of model uncertainties and external disturbances. We first define a sliding surface as follows:

$$s(e,t) = \left(\frac{d}{dt} + \lambda\right)^2 \left(\int_0^t e dr\right) = 0, \tag{5}$$

where $e(t)$ is the state error and λ is a positive constant. By solving the above equation for the control input using Eq. (1), we obtain the following expression for $u(t)$, which is called equivalent control u_{eq}:

$$\begin{aligned}u_{\mathrm{eq}}(t) &= \frac{1}{\hat{g}(\mathbf{x},t)}\cdot(-\hat{f}(\mathbf{x}) + \ddot{x}_d(t) - 2\lambda\dot{e}(t) - \lambda^2 e(t))\\ &= \frac{1}{\hat{g}(\mathbf{x},t)}\cdot\hat{u}(t).\end{aligned} \tag{6}$$

The equivalent control keeps the system states in the sliding surface $s=0$ if the dynamics were exactly known. Hence, if the state is outside the sliding surface, to drive the state to the sliding surface, we choose the control law such that

$$\frac{1}{2}\frac{d}{dt}s^2 \le -\eta|s|, \tag{7}$$

where η is a strictly positive constant and Eq. (7) is called a reaching condition. The control objective is to guarantee that the state trajectory can converge to the sliding surface. It can be proved that the control law

$$u_1(t) = \frac{1}{\hat{g}(\mathbf{x},t)} \cdot [\hat{u}(t) - k \cdot \mathrm{sgn}(s)] \tag{8}$$

with $k \geq \upsilon + \eta$ satisfies the sliding condition (7).[25]

This control law leads to high-frequency control switching and chattering across sliding surface. The value of switching gain k depends on the bounds of system uncertainties. This would require the knowledge on the uncertainty bounds, which are normally difficult to estimate. A large value has to be applied to the control gain when the boundary is unknown. The larger the system uncertainties, the larger the required switching gain to compensate for the effects of uncertainties. Unfortunately, this large control gain may cause chattering on the sliding surface and therefore deteriorate the system performance. The chattering caused by high-frequency switching control activity is highly undesirable and is not acceptable for control of functional electrical stimulation and may excite unmodeled high-frequency plant dynamics which could result in unpredictable instability. To overcome this problem, the SMC strategy deserves special attention because this method provides a systematic approach to maintain asymptotic stability and consistent performance.

A simple method for alleviation of chattering is using a suitable boundary layer around the sliding surface, in which the switching function is approximated by a linear feedback gain when the state trajectory lies within the boundary layer.[25–27] Within the boundary layer, the system no longer behaves as dictated by SMC. By introducing boundary layer, chattering can be reduced, but tracking performance and robustness are compromised.

III. DESIGN OF ADAPTIVE ROBUST CONTROLLER

A. Structure of the ARC

Although SMC has long being known for its capabilities in achieving robust control, however, it also suffers from large control chattering that may excite the unmodeled high-frequency response of the systems due to the discontinuous switching and imperfect implementations. One commonly used method to eliminate the effects of chattering is to replace the switching control law by a saturating approximation within a boundary layer around the sliding surface.[25] Inside the boundary layer, the discontinuous switching function is approximated by a continuous function to avoid discontinuity of the control signals. Even though the boundary layer design can alleviate the chattering phenomenon, these approaches, however, provide no guarantee of convergence to the sliding mode and involve a tradeoff between chattering and robustness and result in the existence of the steady-state error.[31] In order to limit the chattering phenomena and to preserve the main advantages of the original SMC, we propose a new SMC by combining an adaptive nonlinear compensator with SMC. The configuration of the proposed control strategy is schematically depicted in Fig. 1, where u_1 is the SMC input defined in Eq. (8) and u_2 is the output of compensator as the auxiliary control input. Controller output is a function of u_1 and u_2 defined by

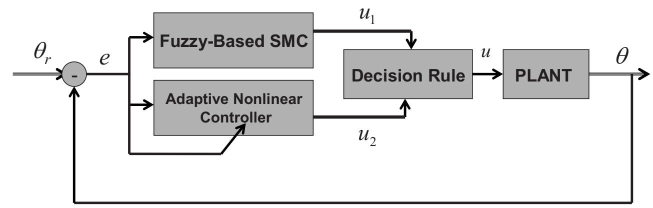

FIG. 1. Configuration of ARC.

$$u = \begin{cases} u_1 & \text{if } |s(e)| > \phi, \\ \xi(e)u_1 + (1-\xi(e))u_2 & \text{if } |s(e)| \leq \phi, \end{cases} \tag{9}$$

where $s(e)$ is a scalar function described in Eq. (2), ϕ is the boundary layer thicknesses, and $\xi(e)$ is a function of error and is adapted by

$$\xi(e) = \frac{|s(e)|}{\phi}. \tag{10}$$

The original SMC structure is retained in the proposed scheme, but the role of adaptive nonlinear compensator becomes more significant as the state trajectory is nearing the sliding surface.

B. Adaptive nonlinear compensator

In order to guarantee the closed-loop stability and minimize the tracking error inside the boundary layer, an adaptive nonlinear compensator is proposed, in which its output is aggregated with the output of SMC when the state trajectory of the system enters in some boundary layer around the sliding surface, i.e.,

$$\begin{aligned} \kappa &= \left(\frac{d}{dt}+1\right)^2 \left(\int_0^t e dr\right), \\ u_2 &= \Lambda\left(e, \dot{e}, \int e \cdot dt\right) = \alpha \cdot \tanh(\varepsilon \cdot (\kappa - \nu)), \end{aligned} \tag{11}$$

where e is the state error. The parameters $\tau = (\alpha, \beta, \nu)$ are adapted online such that the system output x can asymptotically track the desired output x_d. For online adaptation of the parameters $\tau = (\alpha, \beta, \nu)$, the following Lyapunov function V is defined:

$$V = \tfrac{1}{2} \cdot (x - x_d)^2 = \tfrac{1}{2} \cdot e^2. \tag{12}$$

Based on the Lyapunov theorem, the reaching condition is $V \cdot \dot{V} < 0$. If a control input u_2 can be chosen to satisfy this reaching condition, the control system will converge to the origin of the phase plane. By using the following adaptation rule, it was proved that the output tracking error asymptotically converges to zero,[32]

$$\dot{\tau} = -\zeta \cdot e \cdot \frac{\partial u_2}{\partial \tau} \cdot \text{sgn}\left(\frac{\partial x}{\partial u}\right), \tag{13}$$

where $\zeta > 0$ is the learning rate parameter and $\text{sgn}(\cdot)$ is a sign function.

C. Fuzzy-based SMC

To implement the SMC, the nonlinear function $f(\mathbf{x})$ and the control gain $g(\mathbf{x})$ in Eq. (1) should be estimated. In this work, we use the fuzzy logic system (FLS) to approximate the nonlinear function $f(\mathbf{x})$. An interesting and tempting authority of fuzzy logic systems is their capability in approximation of variety types of nonlinear functions. It is proved that certain classes of fuzzy logic systems have universal approximation ability.[33] The fuzzy system uses the fuzzy if-then rules to perform a mapping from an input vector $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in \Re^n$ to an output $O(\mathbf{x}) \in \Re$. The rth fuzzy rule is written as

$$R^r\text{:if } x_1 \text{ is } A_1^r(x_1) \text{ and } x_n \text{ is } A_n^r(x_n), \text{ then } y \text{ is } B^r,$$

where A_i^r and B_i^r are fuzzy sets with membership functions $\mu_{A_i^r}(x_i)$ and $\mu_{B^i}(y)$, respectively, and $\mathbf{x}$ belongs to a compact set. By using the product-inference rule, singleton fuzzifier, and center-average defuzzifier, the output of FLS can be expressed as

$$O(\mathbf{x}) = \frac{\sum_{i=1}^{n_r} \tilde{y}^i \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^{n_r} \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)} = \vartheta^T \psi(\mathbf{x}), \tag{14}$$

where n_r is the number of total fuzzy rules, $\tilde{y}^i$ is the fuzzy singleton for the output in the ith rule, $\mu_{A_j^i}(x_j)$ is the membership function of the fuzzy variable x_j characterized by Gaussian function, $\vartheta = [\tilde{y}^1, \tilde{y}^2, \ldots, \tilde{y}^{n_r}]^T$ is an adjustable parameter vector, and $\psi = [\psi^1, \psi^2, \ldots, \psi^{n_r}]^T$ is a fuzzy basis vector, where ψ^j is defined as

$$\psi^j(\mathbf{x}) = \frac{\left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^{n_r} \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)}. \tag{15}$$

To approximate the nonlinear function estimate $f(\mathbf{x})$ in Eq. (1) using FLS approximator in Eq. (14) and estimate the control gain $g(\mathbf{x}) = g$, adaptive update laws to adjust the parameter vectors ϑ and g need to be developed. In this work, we use the standard recursive least-squares (RLS) algorithm to estimate the parameters.

D. Decentralized ARC

To implement the ARC, the dynamics of the system are presented in a standard canonical form as

$$\begin{aligned} x_1^{(n)} &= f_1(x_1, x_2, \ldots, x_n) + g_1(x_1, x_2, \ldots, x_n)u_1(t) + w_1(t), \\ x_2^{(n)} &= f_2(x_1, x_2, \ldots, x_n) + g_2(x_1, x_2, \ldots, x_n)u_2(t) + w_2(t), \\ &\vdots \\ x_n^{(n)} &= f_n(x_1, x_2, \ldots, x_n) + g_n(x_1, x_2, \ldots, x_n)u_n(t) + w_n(t), \end{aligned} \tag{16}$$

where $u_i(t)$ is the additive input to the system and w_n represents the system parameter uncertainty, unmodeled dynamics, and external disturbances. Instead of designing a MIMO controller, we design a set of independent controllers. Each state variable has its own controller.

Fuzzy modeling approach was used to approximate the nonlinear functions $f_i(x_1, \ldots, x_n)$. Offline identification of the

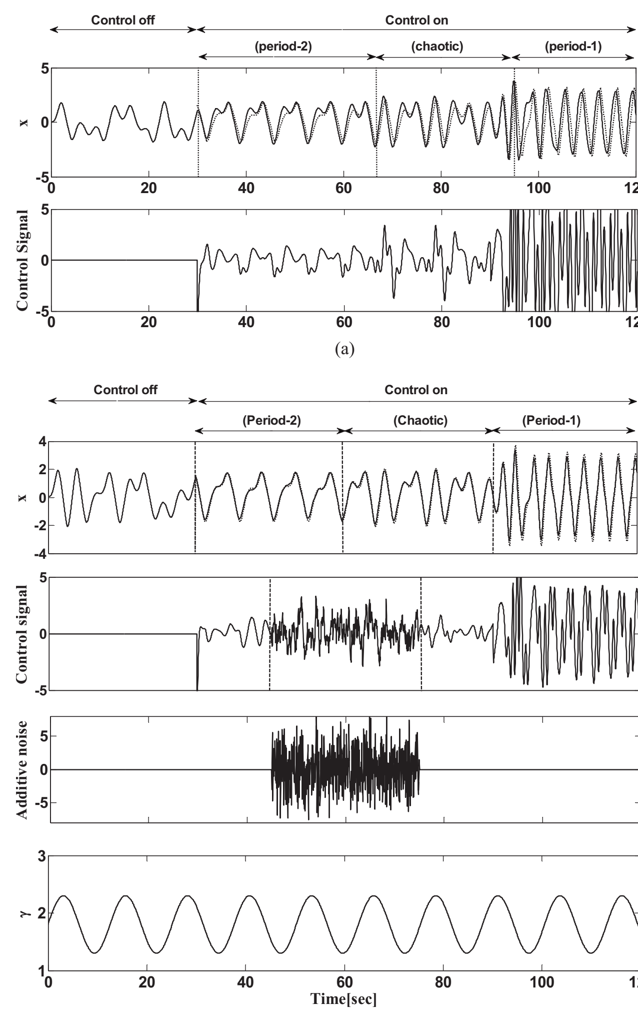

FIG. 2. Controlling the chaotic motion of Duffing's equation to a period-two orbit, period two to a chaotic motion, and chaotic to a period one using (a) conventional SMC ($k=5, \phi=10, \lambda=3$) and (b) proposed ARC ($k=5, \phi=10, \lambda=180, \zeta=0.001$) under the external noise disturbances and time-varying bifurcation parameter. The actual (solid) and desired (dotted) trajectories are shown in top plots of (a) and (b).

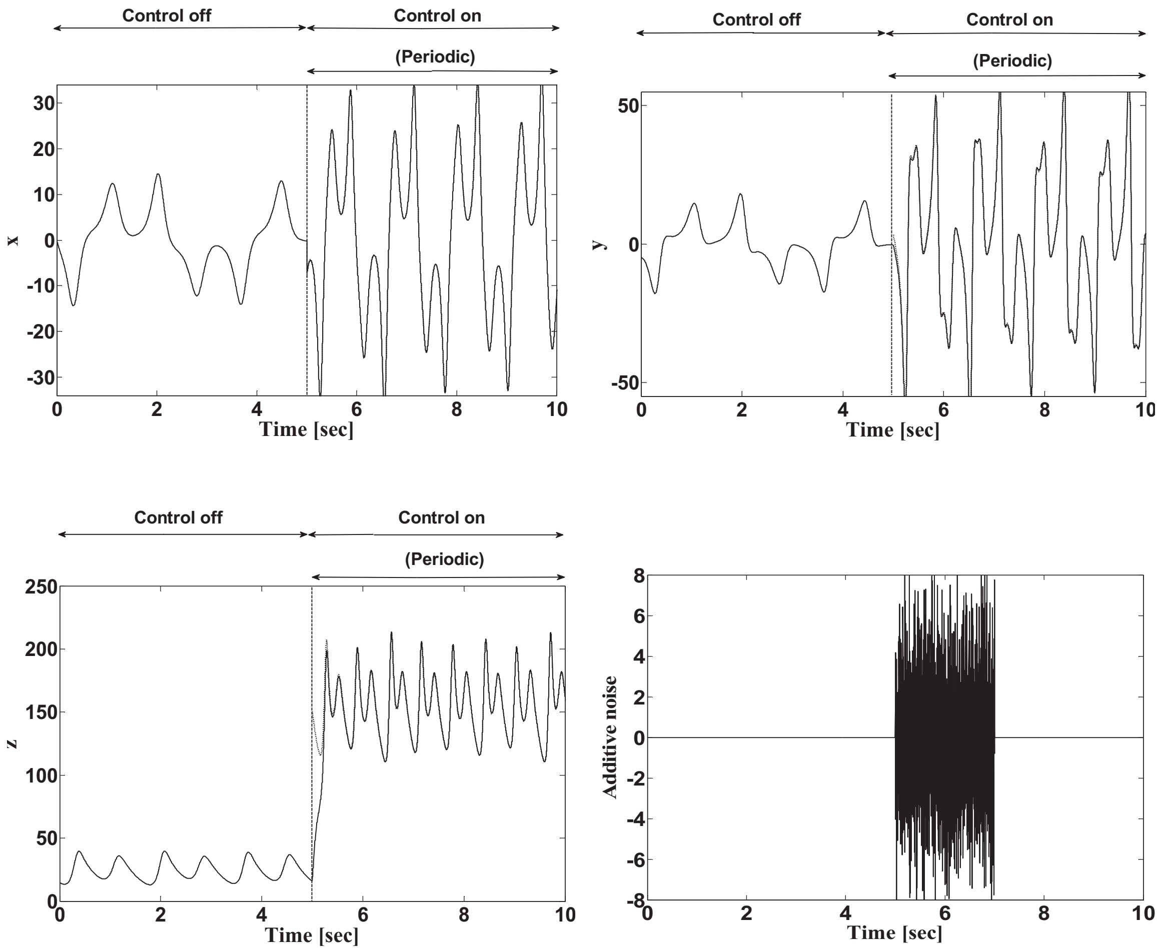

FIG. 3. Converting the chaotic motion of Lorenz dynamics to a desired periodic orbit with additive noise to the state variables of the system using the proposed SMC ($k=5, \phi=10, \lambda=180, \zeta=0.001$).

fuzzy models $f_i(x_1, \ldots, x_n)$ is performed by RLS algorithm using the measured values of the state variables without exogenous inputs and external disturbances.

IV. SIMULATION STUDIES

In this section, we report the simulation results for three representative examples: the second-order nonautonomous Duffing's equation, the third-order continuous-time Lorenz system, and the Rössler system. These are used to verify and demonstrate the effectiveness of the proposed chaos control method.

A. Duffing's equation

Consider the Duffing's equation[5]

$$\ddot{x} + \delta \cdot \dot{x} + \beta \cdot x - x^3 = \gamma \cos(\omega t). \tag{17}$$

The system exhibits very interesting dynamics ranging from periodic to chaotic by using γ as the bifurcation parameter. The solution trajectory of equation is chaotic with $\gamma=1.800$ and 2.100, period two with $\gamma=1.489$, and periodic one with $\gamma=7.000$ when $\delta=0.4$, $\beta=-1.1$, and $\omega=1.8$.

To implement the ARC, the Duffing's equation is presented in a standard canonical form as

$$\ddot{x} = f(x,\dot{x}) + u(t), \tag{18}$$

where $u(t)$ is the additive input to the chaotic system determined by the controller (2). The nonlinear function $f(x,\dot{x})$ is approximated by using a fuzzy logic system described in Sec. III with no additive input. The results of simulation using conventional SMC and proposed method are shown in Fig. 2. The controller was activated at 30 s to convert the chaotic motion with $\gamma=1.800$ to period two starting at 30 s, to chaotic motion with parameter $\gamma=2.100$ starting at 60 s, and to period one at 90 s. It is clearly observed that the high control activity and chattering are due to traditional SMC [Fig. 2(a)]. Moreover, the poor tracking performance is evident. In contrast, an accurate and robust tracking response was achieved to the desired motions. Note that the nonlinear function $f(x,\dot{x})$ was identified offline using the chaotic data obtained with parameter $\gamma=1.800$ and used for all stages of control. Interesting observation is the fast convergence at the instance of changing the desired motion.

To evaluate the performance of controller under time-varying parameter, the bifurcation parameter was changed as the following form:

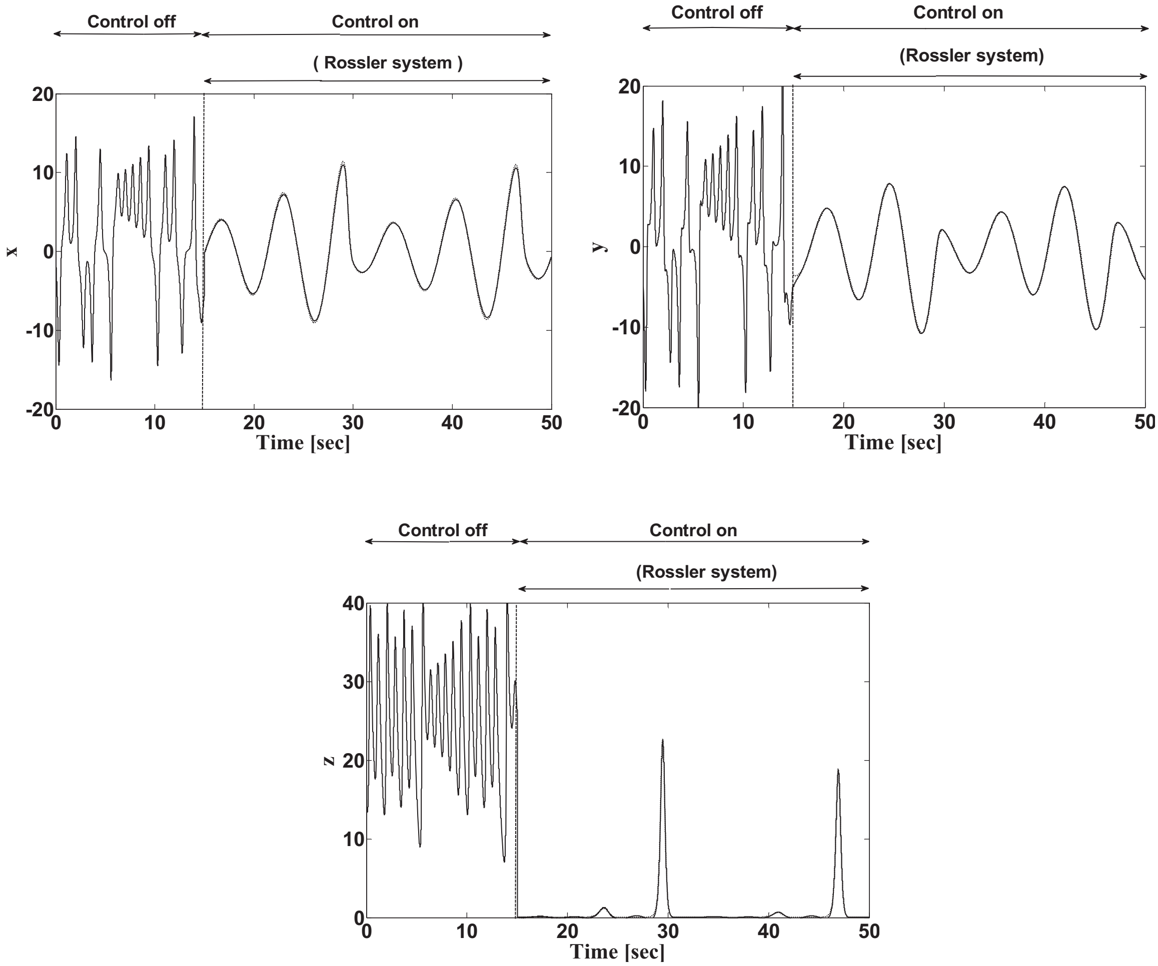

FIG. 4. Converting the chaotic motion of Lorenz dynamics to the Rössler system using the proposed SMC ($k=5, \phi=10, \lambda=180, \zeta=0.001$).

$$\gamma(t) = 1.800 + 0.5\ \sin(0.5t). \tag{19}$$

Moreover, to demonstrate the ability of the proposed control strategy to external noise rejection, an external input with Gaussian distribution (mean 0 and standard deviation 8) was added to the chaotic system. Figure 2(b) shows that an excellent tracking performance can be also achieved under the external noise disturbances and time-varying parameter using the previous identified nonlinear function $f(x,\dot{x})$. Unwanted behavior is perfectly converted to the desired motions (i.e., period one, period two, and chaotic) using the proposed control strategy.

B. Lorenz system

The Lorenz system is described by[34]

$$\dot{x} = \sigma(y-x), \quad \dot{y} = \rho x - y - xz, \quad \dot{z} = -\beta z + xy, \tag{20}$$

where $\sigma=10$, $\beta=8/3$, and $\rho=28$.

For each state variable, an ARC is designed separately. To implement the ARC, the Lorenz system is presented in the standard canonical form as follows:

$$\begin{gathered}\dot{x} = f_1(x,y,z) + u_1(t), \quad \dot{y} = f_2(x,y,z) + u_2(t), \\ \dot{z} = f_3(x,y,z) + u_3(t),\end{gathered} \tag{21}$$

where $u_1(t)$, $u_2(t)$, and $u_3(t)$ are the controllers. The nonlinear functions $f(x,y,z)$ were identified using the data obtained with parameters $\sigma=10$, $\beta=8/3$, and $\rho=28$ without additive inputs. Figure 3 shows the results of controlling the Lorenz dynamics with parameters $\sigma=10$, $\beta=8/3$, and $\rho=28$. The controller is switched on at 5 s to convert the chaotic motion to a desired period orbit of the given system. Moreover, a disturbance with Gaussian distribution (mean 0 and standard deviation 8) was added to the chaotic system as follows:

$$\begin{gathered}\dot{x} = \sigma(y-x) + v_1, \quad \dot{y} = \rho x - y - xz + v_2, \\ \dot{z} = -\beta z + xy + v_3.\end{gathered} \tag{22}$$

It is observed that the controller is able to convert the chaotic motion to the periodic orbit even in the presence of external noise disturbance.

Figure 4 shows the result of a typical chaos-to-chaos control. The controller was switched on at 15 s to convert the Lorenz dynamics to a different chaotic motion, i.e., the Rössler system,[35] as follows:

$$\dot{x} = -y - z, \quad \dot{y} = x + a \cdot y, \quad \dot{z} = b + z \cdot (x - c).$$

The Rössler system is chaotic for $a=0.2$, $b=0.2$, and $a=5.7$. From the simulation results, it can be seen that a perfect tracking response can be obtained even under the changing the structure of chaotic system.

V. CONCLUSIONS

In this paper, a robust control strategy incorporating the SMC and adaptive control has been proposed for controlling the chaotic dynamics. The controller is constructed such that the tracking error exponentially converges to a small region around zero. Simulation studies on the second-order nonautonomous Duffing's equation and the third-order continuous-time Lorenz system demonstrated the exceptional performance and robustness of the control system against external noise disturbances and system parameter variations during converting the chaotic motion to a periodic orbit and chaos-to-chaos control. An important observation is the fast convergence of the control system. The conversion of unwanted trajectory to the desired motions is very fast. The method does not require any knowledge about the equation of system dynamics. To design the controller, a model of chaotic system should be first identified. However, the model does not require to be accurate.

[1] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[2] X. Yu, G. Chen, Y. Xia, Y. Song, and Z. Cao, IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. **48**, 930 (2001).
[3] M. Ding, W. Yang, V. In, W. L. Ditto, M. L. Spano, and B. Gluckman, Phys. Rev. E **53**, 4334 (1996).
[4] H. G. Schuster, *Handbook of Chaos Control: Foundations and Applications* (Wiley, New York, 1999).
[5] G. Chen and X. Dong, IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. **40**, 591 (1993).
[6] A. Loria, E. Panteley, and H. Nijmeijer, IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. **45**, 1252 (1998).
[7] M. T. Yassen, Chaos, Solitons Fractals **15**, 271 (2003).
[8] R. Wang and Z. Jing, Chaos, Solitons Fractals **21**, 201 (2004).
[9] H. Guo, S. Lin, and J. Liu, Phys. Lett. A **351**, 257 (2006).
[10] W. Chang, Inf. Sci. (N.Y.) **171**, 261 (2005).
[11] T.-Y. Chiang, M.-L. Hung, J.-J. Yan, Y.-S. Yung, and J.-F. Chang, Chaos, Solitons Fractals **34**, 437 (2007).
[12] H. Wang, Z.-Z. Han, Q.-Y. Xie, and W. Zhang, Commun. Nonlinear Sci. Numer. Simul. **14**, 2728 (2009).
[13] W. J. Freeman, Int. J. Bifurcation Chaos Appl. Sci. Eng. **2**, 451 (1992).
[14] T. R. Chay, Y. S. Fan, and Y. S. Lee, Int. J. Bifurcation Chaos Appl. Sci. Eng. **5**, 595 (1995).
[15] W. J. Freeman and G. Viana di Prisco, Proceedings of the First Trieste Meeting on Brain Theory, 1984 (unpublished), pp. 1–4.
[16] W. J. Freeman, Biol. Cybern. **35**, 221 (1979).
[17] W. J. Freeman, Biol. Cybern. **56**, 139 (1987).
[18] W. J. Freeman, IEEE Trans. Circuits Syst. **35**, 781 (1988).
[19] A. Babloyantz, J. M. Salazar, and G. Nicolis, Phys. Lett. **111**, 152 (1985).
[20] J. P. Pinj, J. V. Nerveen, A. Noest, and F. H. Lopes da Silva, Electroencephalogr. Clin. Neurophysiol. **79**, 371 (1991).
[21] D. Gallez and A. Babloyantz, Biol. Cybern. **64**, 381 (1991).
[22] L. Pezard, J.-L. Nandrino, B. Renault, F. El Massioui, J.-F. Allilaire, and J. Müller, Biol. Psychiatry **39**, 991 (1996).
[23] K. Lehnertz and C. E. Elger, Clin. Neurophysiol. **95**, 108 (1995).
[24] K. Lehnertz and C. E. Elger, Phys. Rev. Lett. **80**, 5019 (1998).
[25] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control* (Prentice-Hall, Englewood Cliffs, NJ, 1991).
[26] K. D. Young, V. I. Utkin, and Ü. Őzgüner, IEEE Trans. Control Syst. Technol. **7**, 328 (1999).
[27] J. Guldner and V. I. Utkin, Proceedings of the 14th International Symposium of Mathematical Theory of Networks and Systems, 2000 (unpublished), pp. 27–32.
[28] Z. Wu and P. Yu, IEEE Trans. Autom. Control **51**, 1019 (2006).
[29] G. Chen and X. Dong, Int. J. Bifurcation Chaos Appl. Sci. Eng. **3**, 1363 (1993).
[30] H. Seraji and T. Bajd, IEEE Trans. Rob. Autom. **5**, 183 (1989).
[31] M.-M. Ebrahimpour and A. Erfanian, IEEE Trans. Biomed. Eng. **55**, 2842 (2008).
[32] W.-D. Chang, R.-C. Hwang, and J.-G. Hsieh, IEEE Trans. Syst. Man Cybern., Part C Appl. Rev. **32**, 517 (2002).
[33] L.-X. Wang, *A Course in Fuzzy Systems and Control* (Prentice-Hall, Englewood Cliffs, NJ, 1997).
[34] C. Sparrow, IEEE Trans. Circuits Syst. **30**, 533 (1983).
[35] O. E. Rössler, Phys. Lett. **57**, 397 (1976).