General Lossy External Cloak with New Structures

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Abstract
In this article we consider loss for all parts of external cloaks that are achieved by the different transformation functions. Each cloak has a different threshold for loss. We compare the simulation results with each other to find the best function for external cloaking.

1. Introduction
Controlling the electromagnetic waves for cloaking an object is an interesting field, which has recently received much attention[1-5]. Firstly, Pendry was capable to achieve a new method for controlling the electromagnetic waves with coordinate transformation [6-7]. All functions that satisfy transform conditions may be used [8]. One method to achieve the invisibility cloak is using the coordinate transformation. A new kind of cloaking based on this method is external cloak which make invisible an object in special distance out of the cloak [9-12]. The main idea for this design is a concept of complementary media [9, 13-16]. Complementary media is a material with negative refraction index which cancels the optical effects of any external object placed in a specific distance [17-20].

In this article, we consider several new coordinate transformation functions in linear, exponential and square forms for designing external cloak. Most of the works in this field especially in external cloak consider lossless media for cloak, but all the materials naturally are lossy, thus lossy environments were considered for inner core and outer shell of the cloaks and the simulation results were compared. The results indicate that the acceptable loss of that inner core or outer shell of cloak to be capable to cloak the external region depends on the function of coordinate transformation that was used.

2. Formulation and the structures
The purpose of external cloak is cloaking the effects of an object that lies out of the cloak[7]. In this section we propose new functions for transformation. The simulation results are shown in figure 2.

According to the theory of coordinate transformation when we map one space to another one, the permittivity ($\varepsilon'$) and permeability ($\mu'$) of the transformed space ($r'$) are obtained from the following formulas:

$$\varepsilon' = \frac{A\varepsilon A^T}{\det(A)} \tag{1}$$

$$\mu' = \frac{A\mu A^T}{\det(A)} \tag{2}$$

Where, $\varepsilon$ and $\mu$ are the permittivity and permeability of old space ($r$) respectively. $A$ is the Jacobian of transformation matrix and the components are obtained by $A_{ij} = \frac{\partial x_i}{\partial x_j}$ [16, 21]. Complementary medium is the material with negative permittivity and permeability of the outer space which have the special property that if an object is placed between 0 to L it cancels the optical effect as there is nothing[15]. (Figure 1)

The main idea to achieve material parameters is combining the theory of complementary media and coordinate transformation. The first step is transformation of the region $a<r<b$ on to the region $b<r'<c$, this transformation result $a<r<b$ with negative refractive index so the region $a<r''<c$ is optically void[7-9]. Any transformation function that satisfies these transformation conditions is acceptable[6].

Hence we use three functions in order to transform $a<r<b$ to $b<r'<c$ as:

$$f\left(r'\right) = [(r' - b)(c - b)/(a - b)] \ast b \tag{3}$$

$$f\left(r'\right) = (e^{-r'} / e^{-r})[(r' - b)(c - b)/(a - b)] \ast b \tag{4}$$

Figure 1. (a) a slab of complementary medium which makes the external object invisible by canceling the optical effects of it. (b) a cylindrical form complementary medium which has the same effect as (a).